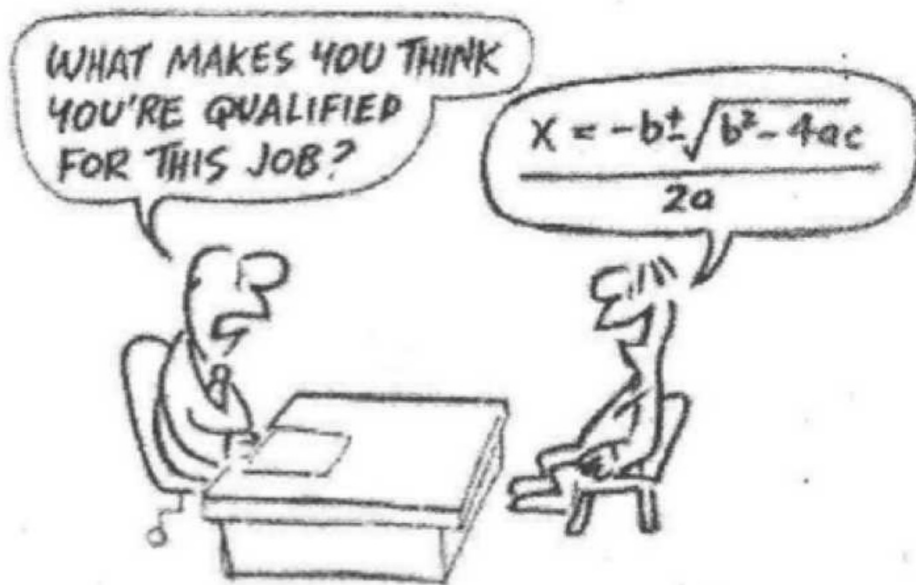


1.7 Solving Quadratic Equations Using CTS
1.8 Quadratic Formula



Solving Quadratics By CTS

*Use CTS to solve a quadratic equation when...

"b" value is even

ex: Solve.

a) $x^2 - 4x - 10 = 0$

b) $x^2 - 14x + 103 = 0$

$$\text{b) } x^2 - 14x + 103 = 0$$

$$c) -2x^2 + 4x - 17 = 0$$

Solving Quadratics Using the Quadratic Formula

Let $a, b, c \in \mathbb{R}$ such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are:

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Use the Quadratic Formula to solve a quadratic equation when...

the values of $a, b, + c$ are small
and the quadratic is not factorable.

ex: Solve.

a) $x^2 + 3x = 2$

$$x^2 + 3x - 2 = 0$$

$$\begin{aligned} a &= 1 \\ b &= 3 \\ c &= -2 \end{aligned}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)} = \frac{-3 \pm \sqrt{17}}{2}$$

or

$$-\frac{3}{2} \pm \frac{\sqrt{17}}{2}$$

b) ~~$x^2 + 4x - 5 = 0$~~

$$-x^2 + 5x - 4 = x + 1$$

$$-(-x^2 + 4x - 5 = 0)$$

$$x^2 - 4x + 5 = 0$$

$$a = 1$$

$$b = -4$$

$$c = 5$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-4}}{2}$$

$$x = \frac{4 \pm 2i}{2} = \frac{4}{2} + \frac{2i}{2} \\ = 2 \pm i$$

$$c) \quad x^2 = 6x - 4$$

$$x^2 - 6x + 4 = 0$$

$$a = 1$$

$$b = -6$$

$$c = 4$$

$$x = \frac{6 \pm \sqrt{36 - 4(1)(4)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2}$$

$$= 3 \pm \sqrt{5}$$

Picking A Method

a.) $2x^2 - 6x + 1 = 0$

quad. form.

b.) $-2x^2 - 7x + 15 = 0$

factor

c.) $6 - 2(x+1)^2 = 0$

square root

d.) $3x^2 - 12x - 14 = 0$

CTS

$$a.) \quad 2x^2 - 6x + 1 = 0$$

$$a = 2$$

$$b = -6$$

$$c = 1$$

$$x = \frac{6 \pm \sqrt{36 - 4(2)(1)}}{4}$$

$$x = \frac{6 \pm \sqrt{28}}{4} = \frac{6 \pm 2\sqrt{7}}{4}$$

$$\frac{6}{4} \pm \frac{2\sqrt{7}}{4} = \left(\frac{3}{2} \pm \frac{\sqrt{7}}{2} \right)$$

$$b.) -2x^2 - 7x + 15 = 0$$

$$2x^2 + 7x - 15 = 0$$

$$(2x - 3)(x + 5) = 0$$

↓

$$x = \frac{3}{2}, -5$$

$$\begin{array}{l} 21 \\ -35 \end{array}$$

$$c.) \quad 6 - 2(x+1)^2 = 0$$

$$-2(x+1)^2 = -6$$

$$\sqrt{(x+1)^2} = \sqrt{3}$$

$$|x+1| = \sqrt{3}$$

$$x+1 = \pm\sqrt{3}$$

$$x = -1 \pm \sqrt{3}$$

$$d.) \quad 3(x^2 - 4x + 4) - 12 - 14 = 0$$

$$3(x-2)^2 = 26$$

$$\sqrt{(x-2)^2} = \sqrt{\frac{26}{3}} \cdot \sqrt{3}$$

$$|x-2| = \frac{\sqrt{78}}{3}$$

$$x = 2 \pm \frac{\sqrt{78}}{3}$$

$13 \cdot 2 \cdot 3$

ex: Determine which method is best to solve each quadratic equation. Do not repeat a method. DO NOT SOLVE.

a)

1. $x^2 + 6x - 3 = 0$

2. $x^2 + 6x + 5 = 0$

3. $2(x+1)^2 - 4 = 0$

4. $x^2 + 2x + 5 = 0$

ex: Determine which method is best to solve each quadratic equation. Do not repeat a method. DO NOT SOLVE.

a)

1. $x^2 + 6x - 3 = 0$ *CTS or Quad* $x = -3 \pm 2\sqrt{3}$

2. $x^2 + 6x + 5 = 0$ *Factor* $x = -1, -5$

3. $2(x+1)^2 - 4 = 0$ *Sq. root* $x = -1 \pm \sqrt{2}$

4. $x^2 + 2x + 5 = 0$ *CTS or Quad* $x = -1 \pm 2i$

b)

1. $14x^2 - 21x = 0$

2. $x^2 + 3x - 1 = 0$

3. $2x^2 - 8x + 5 = 0$

4. $x^2 - 80 = 0$

b)

factor 1. $14x^2 - 21x = 0$ $x = 0, \frac{3}{2}$

Quad
f/m 2. $x^2 + 3x - 1 = 0$ $x = \frac{-3 \pm \sqrt{13}}{2}$

CTs 3. $2x^2 - 8x + 5 = 0$ $x = 2 \pm \frac{\sqrt{6}}{2}$

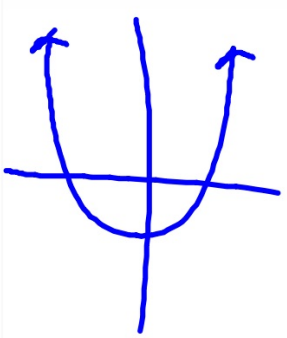
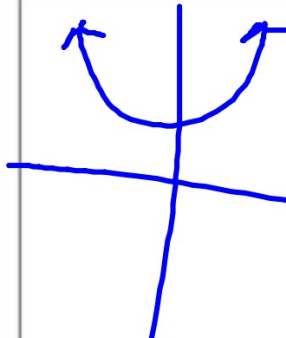
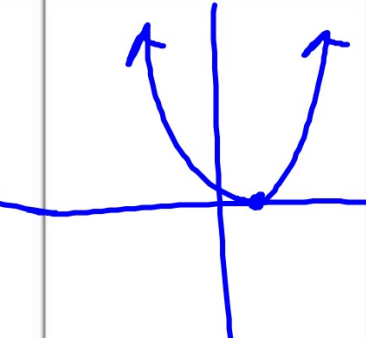
Sq.
root 4. $x^2 - 80 = 0$ $x = \pm 4\sqrt{5}$

The Discriminant:

- In the quadratic formula, the expression $b^2 - 4ac$ is called the discriminant.
- The discriminant is used to determine the the nature of the roots (or solutions) of the quadratic equation

$$D = b^2 - 4ac$$

Using The Discriminant:

Value of discriminant	$D > 0$	$D < 0$	$D = 0$
Number of solutions	2	2	1
Type of solutions	real	imaginary	real
Graph of $y = ax^2 + bx + c$			

ex: Find the discriminant and give the number and type of solutions of the equation.

a) $x^2 - 8x + 13 = -4$

$$x^2 - 8x + 17 = 0$$

$$a = 1 \quad D = (-8)^2 - 4(1)(17)$$

$$b = -8 \quad = 64 - 68$$

$$c = 17$$

-4 ; 2 imaginary solutions.

$$b) x^2 - 8x + 16 = 0$$

$$a = 1$$

$$b = -8$$

$$c = 16$$

$$D = (-8)^2 - 4(1)(16)$$

$$= 64 - 64$$

$$= 0$$

1 real solution

$$c) 8x^2 - 2x + 1 = x^2 + 6$$

$$7x^2 - 2x - 5 = 0$$

factorable!

$$\begin{aligned} a &= 7 \\ b &= -2 \\ c &= -5 \end{aligned}$$

$$D = (-2)^2 - 4(7)(-5)$$

$$= 4 + 140$$

$$= 144; \text{ 2 real}$$

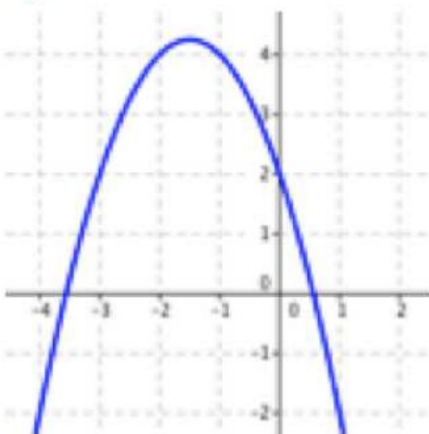
perfect square

$$\begin{array}{l} 7 \quad | \\ \hline 5 \quad -1 \end{array}$$

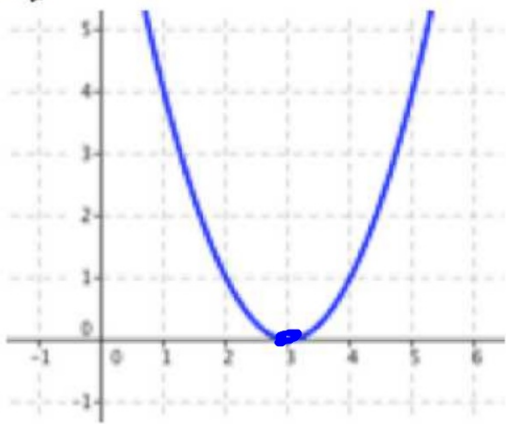
$$(7x+5)(x-1) = 0$$

ex: The graph of $y = ax^2 + bx + c$ and the solutions of $ax^2 + bx + c = 0$ are given. Determine if the discriminant is positive, negative, or zero. Explain your reasoning.

a)



b)



zero

$$c) x = 2 \pm 3i$$

D is negative

ex: Consider the quadratic equation: $3x^2 + 12x + c = 0$
Find all values of c for which the equation has...

a) two real solutions

$$D > 0$$

$$b^2 - 4ac > 0$$

$$144 - 12c > 0$$

$$c < 12$$

b) one real solution

$$b^2 - 4ac = 0$$
$$144 - 12c = 0$$
$$c = 12$$

c) two imaginary solutions

$$b^2 - 4ac < 0$$
$$144 - 12c < 0$$
$$c > 12$$