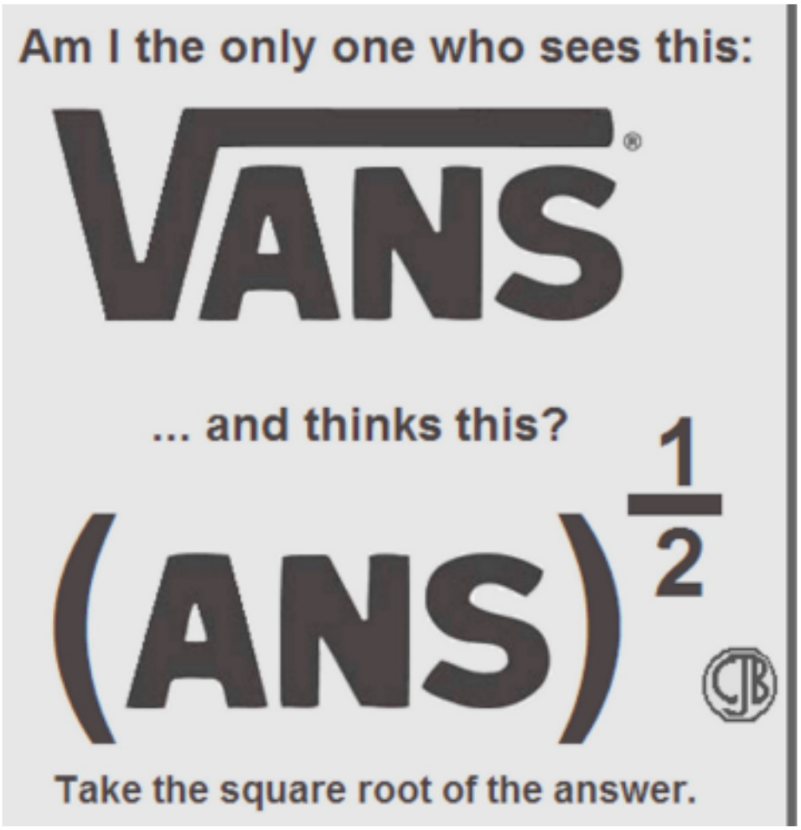
A2: Rational Exponents



Rational Exponents

$$\boxed{x^{\frac{e}{i}} = \sqrt[i]{x^e}}$$

$9^{1/2} = \sqrt[2]{9^1}$

$\sqrt{9}$

3

Where:

- e is called the exponent
- i is called the index

ex: Rewrite in radical form.

a) $4^{3/5}$ $\sqrt[5]{4^3}$

b) $27^{4/3}$ $\sqrt[3]{27^4}$

c) $9^{5/2}$ $\sqrt[2]{9^5}$

ex: Rewrite in radical form.

d) $-4^{1/2}$

e) $(-3)^{5/3}$

f) $2 \cdot 7^{3/4}$

ex: Rewrite in exponential form.

a) $\sqrt[7]{21} = 21^{1/7}$

b) $\sqrt{8^3} = 8^{3/2}$

$9^{0.5} = 9^{1/2}$

$= \sqrt{9}$

$= 3$

c) $\sqrt[3]{-9} = (-9)^{1/3}$

ex: Evaluate. If no real value exists, write "nonreal."

a) $9^{3/2} = \sqrt{9^3} = (\sqrt{9})^3 = 3^3 = 27$

b) $81^{3/4} = (\sqrt[4]{81})^3 = 3^3 = 27$

c) $4^{5/2} = (\sqrt[2]{4})^5 = 2^5 = 32$

ex: Evaluate. If no real value exists, write "nonreal."

d) $8^{4/3}$ $= \left(\sqrt[3]{8}\right)^4 = 2^4 = 16$

e) $125^{2/3}$ $= \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

f) $32^{3/5}$ $= \left(\sqrt[5]{32}\right)^3 = 2^3 = 8$

ex: Evaluate. If no real value exists, write "nonreal."

g) $(-216)^{2/3}$ $= -\left(\sqrt[3]{216}\right)^2 = -(6)^2 = -36$

h) $(-16)^{5/2}$ nonreal

even index

i) $-4^{3/2}$ $= -1 \cdot 4^{3/2} = -1 \cdot 8 = -8$

$\left(\sqrt[2]{4}\right)^3$

2^3

ex: Evaluate. If no real value exists, write "nonreal."

j) $2\cdot\left(32^{3/5}\right) = 2\left(\sqrt[5]{32}\right)^3 = 2^1\cdot 2^3 = 2\cdot 8 = 16$

2^4

k) $\left(\frac{81}{16}\right)^{3/4} = \frac{\left(\sqrt[4]{81}\right)^3}{\left(\sqrt[4]{16}\right)^3} = \frac{3^3}{2^3} = \frac{27}{8}$

l) $16^{-3/4} = \frac{1}{16^{3/4}} = \frac{1}{\left(\sqrt[4]{16}\right)^3} = \frac{1}{8}.$

ex: Evaluate. If no real value exists, write "nonreal."

m) $9^{-5/2} = \frac{1}{9^{5/2}} = \frac{1}{(\sqrt{9})^5} = \frac{1}{3^5} = \frac{1}{243}$

n) $-8^{-2/3} = -1 \cdot \frac{1}{8^{2/3}} = -1 \cdot \frac{1}{(\sqrt[3]{8})^2} = \frac{-1}{4}$

o) $\left(\frac{81}{4}\right)^{-3/2} = \left(\frac{4}{81}\right)^{3/2} = \frac{(\sqrt{4})^3}{(\sqrt{81})^3} = \frac{8}{729}$

p) $\left(\frac{27}{8}\right)^{-4/3} = \left(\frac{8}{27}\right)^{4/3} = \frac{(\sqrt[3]{8})^4}{(\sqrt[3]{27})^4} = \frac{16}{81}$