

$$15. \frac{2}{(4+3\sqrt{2})} \cdot \frac{(4-3\sqrt{2})}{(4-3\sqrt{2})} \quad \text{F\cancel{O}I\cancel{L}} \\ = \quad =$$

$$\frac{8-6\sqrt{2}}{16-9\sqrt{4}} = \frac{8-6\sqrt{2}}{-2}$$

$$\downarrow \\ 16-18$$

$$\frac{8}{-2} - \frac{6\sqrt{2}}{-2}$$

$$\textcircled{-4 + 3\sqrt{2}}$$

$$1.) \quad 5\sqrt{15} \cdot 3\sqrt{20}$$

$$15\sqrt{15 \cdot 20}$$

$$\rightarrow 15\sqrt{300}$$

$$15\overset{10}{\sqrt{100}} \cdot \sqrt{3}$$

$$150\sqrt{3}$$

$$16.) \frac{3}{(3-\sqrt{5})} \cdot \frac{(3+\sqrt{5})}{(3+\sqrt{5})}$$

$$\frac{9+3\sqrt{5}}{9-5}$$

$$\frac{9+3\sqrt{5}}{4}$$

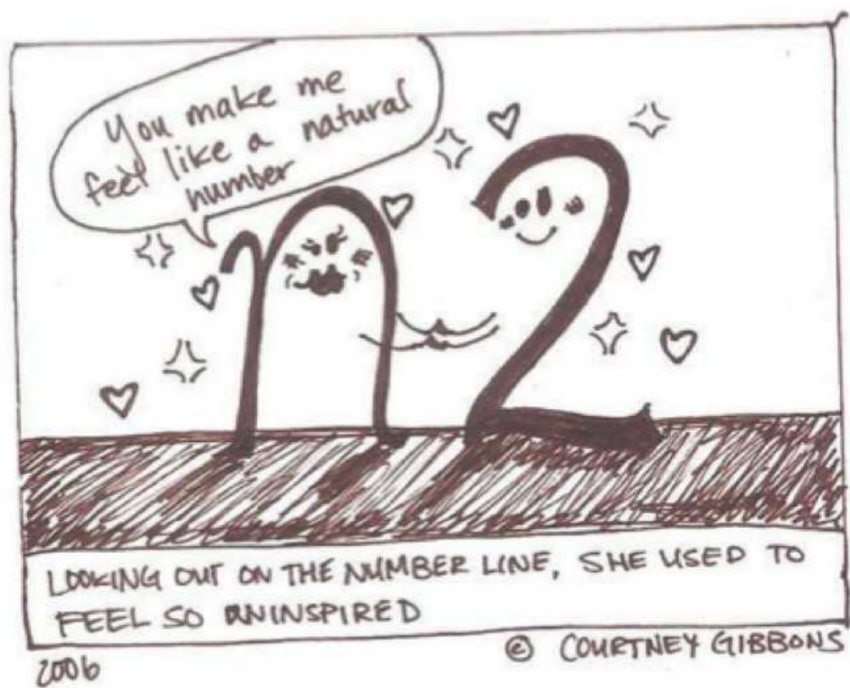
$$12.) \quad (3\sqrt{3} + \sqrt{5})(4\sqrt{2} + \sqrt{5})$$

$$12\sqrt{6} + 3\sqrt{15} + 4\sqrt{10} + 5$$

$$20.) \frac{\sqrt[5]{3}}{\sqrt[5]{8}} \cdot \frac{\sqrt[5]{4}}{\sqrt[5]{4}} = \frac{\sqrt[5]{12}}{2}$$

$$\begin{aligned}
 17.) \quad \frac{2}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} &= \frac{2\sqrt[3]{4}}{\sqrt[3]{8}} \\
 &= \frac{\cancel{2}\sqrt[3]{4}}{\cancel{2}}
 \end{aligned}$$

A2 Simplifying nth Roots with Variables



ex: Simplify. If no real value exists, write "nonreal."

$$\begin{aligned}\text{a) } \sqrt[3]{40} &= \sqrt[3]{8} \cdot \sqrt[3]{5} \\ &= 2\sqrt[3]{5}\end{aligned}$$

$$\text{b) } -\sqrt[4]{162} = -\sqrt[4]{81} \sqrt[4]{2} = -3\sqrt[4]{2}$$

$$\text{c) } \sqrt[5]{-250} = -\sqrt[5]{250}$$

ex: Simplify. If no real value exists, write "nonreal."

d) $\frac{5}{\sqrt[3]{25}} \cdot \frac{\sqrt[3]{8}}{\sqrt[3]{5}}$

$$\frac{\cancel{5} \sqrt[3]{5}}{\cancel{5}}$$

$$\sqrt[3]{5}$$

Simplifying nth Roots Involving Variables

$$\frac{\sqrt[3]{8}}{2}$$

$$x^2 \cdot x^3 = x^5$$

ex: Simplify. Assume all variables are positive.

$$\begin{aligned} \text{a) } \sqrt[3]{x^4} &= \sqrt[3]{x^3} \cdot \sqrt[3]{x^1} \\ &= x \sqrt[3]{x} \end{aligned}$$

ex: Simplify. Assume all variables are positive.

$$\begin{aligned} \text{b) } \sqrt[5]{x^{22}} &= \sqrt[5]{\underbrace{x^5 \cdot x^5 \cdot x^5 \cdot x^5 \cdot x^2}} \\ &= x^4 \sqrt[5]{x^2} \qquad \sqrt[5]{x^{20} \cdot x^2} = x^4 \sqrt[5]{x^2} \end{aligned}$$

$$\text{c) } \sqrt[7]{x^{17}} = \sqrt[7]{x^{14} \cdot x^3} = x^2 \sqrt[7]{x^3}$$

$$\begin{aligned} \text{d) } \sqrt[9]{x^{21}} &= \sqrt[9]{x^{18} \cdot x^3} = x^2 \sqrt[9]{x^3} \\ &\qquad \sqrt[4]{x^4} = x \\ &\qquad \sqrt[4]{x^8} = x^2 \end{aligned}$$

ex: Simplify. Assume all variables are positive.

e) $\sqrt[3]{16x^4y^6z^2}$ $y^2 \sqrt[3]{8 \cdot 2 \cdot x^3 \cdot x^1 z^2}$
 $2xy^2 \sqrt[3]{2xz^2}$

f) $\sqrt[5]{-96xy^{10}z^{14}}$ $= -\sqrt[5]{32 \cdot 3 \cancel{xy^{10}} (z^{10} z^4)}$
 $-2y^2z^2 \sqrt[5]{3xz^4}$

ex: Simplify. Assume all variables are positive.

$$g) \sqrt[2]{x} = \sqrt{x}$$

$$\sqrt{16} = 4$$

$$h) \sqrt[2]{x^4} = x^2$$

$$\sqrt[2]{2^4} = 2^2$$

$$i) \sqrt[2]{x^6}$$

$$x^3$$

ex: Simplify. Assume all variables are positive.

$$j) \sqrt[4]{x^8} = x^2$$

$$k) \sqrt[4]{x^5} = \sqrt[4]{x^4 \cdot x^1} = x^1 \sqrt[4]{x}$$

ex: Simplify. Assume all variables are positive.

$$l) \sqrt[6]{x^6 y^{12} z^{20}} = xy^2 \sqrt[6]{z^{18} z^2} = xy^2 z^3 \sqrt[6]{z^2}$$

$$m) \sqrt[4]{48x^3 y^{12} z^{24}} = y^3 z^6 \sqrt[4]{16 \cdot 3x^3} \\ 2y^3 z^6 \sqrt[4]{3x^3}$$

ex: Simplify. Assume all variables are positive.

$$\begin{aligned} \text{n) } \sqrt{200x^3y^4z} &= y^2 \sqrt{100 \cdot 2x^2 \cdot x^1 z} \\ &= 10xy^2 \sqrt{2xz} \end{aligned}$$

$$\sqrt{x^2} = x$$

$$\sqrt{x^4} = x^2$$

$$\text{o) } \sqrt[3]{-16xy^3z^{10}}$$

$$-y \sqrt[3]{8 \cdot 2x \cancel{z^3} \cdot z^1}$$

$$-2yz^3 \sqrt[3]{2x2}$$

Review

Simplify.

$$\sqrt[4]{-81}$$

nonreal

$$\begin{aligned} &\sqrt[3]{-81} \\ &- \sqrt[3]{27 \cdot 3} \\ &- 3\sqrt[3]{3} \end{aligned}$$

Review

Simplify.

$$\frac{10}{\sqrt[5]{-16}}$$

$$\frac{10}{-\sqrt[5]{16}} \cdot \frac{\sqrt[5]{2}}{\sqrt[5]{2}}$$

$$\frac{10\sqrt[5]{2}}{-2} = -5\sqrt[5]{2}$$

Review

Between which two consecutive integers does the expression lie?

$$\sqrt[3]{-7}$$

Review

Between which two consecutive integers does the expression lie?

$$\sqrt[3]{2}$$

$$\sqrt[3]{1} < \sqrt[3]{2} < \sqrt[3]{8}$$

$$\boxed{1, 2}$$

$$\begin{aligned} 1^4 &= 1 \\ 2^4 &= 16 \\ 3^4 &= 81 \\ 4^4 &= 256 \end{aligned}$$

$$\sqrt[4]{40}$$

$$\begin{array}{c} \sqrt[4]{16} \\ \downarrow \\ 2 \end{array} < \sqrt[4]{40} < \begin{array}{c} \sqrt[4]{81} \\ \downarrow \\ 3 \end{array}$$