n^{th} Roots - Multiplying, Dividing, Rationalizing

$$\frac{\sqrt{2}}{2} = \sqrt{}$$

HW:

Operations with n^{th} Roots: Multiplication

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers,

then $\sqrt[n]{a} \cdot \sqrt[n]{b} =$ $\sqrt[n]{a \cdot b}$.

*The indexes must be the same when multiplying.

ex: Can you simplify the product of the radical expressions? Explain.

a) $\sqrt[3]{6} \cdot \sqrt{2}$

no, different indexes

b) $\sqrt[3]{-4} \cdot \sqrt[3]{2}$

$\sqrt[3]{-8}$

-2

ex: Multiply, if possible. Then simplify.

a) $\sqrt[3]{5} \cdot \sqrt[3]{-25}$

$\sqrt[3]{-125}$

-5

ex: Multiply, if possible. Then simplify.

b) $\sqrt[4]{18} \cdot \sqrt[4]{6}$

$$\boxed{\sqrt[4]{108}}$$

ex: Multiply, if possible. Then simplify.

c) $\sqrt[4]{8} \cdot \sqrt[3]{32}$ $\longrightarrow$ $\sqrt[4]{8} \cdot \sqrt[3]{8} \cdot \sqrt[3]{4}$

Already simplified

$2\sqrt[4]{8}\sqrt[3]{4}$

ex: Multiply, if possible. Then simplify.

d) $\sqrt[3]{6} \cdot \sqrt[3]{16}$

$\sqrt[3]{96}$

$\sqrt[3]{8} \cdot \sqrt[3]{12}$

$\downarrow$

$2\sqrt[3]{12}$

$\sqrt[3]{2 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$

$\sqrt[3]{12} \cdot \sqrt[3]{8}$

$2\sqrt[3]{12}$

ex: Multiply, if possible. Then simplify.

e) $2\sqrt[3]{4} \cdot 7\sqrt[3]{80}$

$14\sqrt[3]{320}$

$14\sqrt[3]{64}\sqrt[3]{5}$

$14 \cdot 4\sqrt[3]{5}$

$56\sqrt[3]{5}$

8

27

64 ✓

~~125~~

~~216~~

$14\overset{2}{\cancel{\sqrt[3]{8}}}\sqrt[3]{40}$

Operations with n^{th} Roots: Division

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers,

then $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \underline{\sqrt[n]{\dfrac{a}{b}}}$.

*The indexes must be the same when dividing.

ex: Divide, if possible. Then simplify.

a) $\dfrac{\sqrt{18}}{\sqrt{2}}$

$\sqrt{9}$

3

ex: Divide, if possible. Then simplify.

b) $\dfrac{\sqrt[3]{162}}{\sqrt[3]{3}}$

$\sqrt[3]{54}$

$\sqrt[3]{27} \cdot \sqrt[3]{2}$

$\downarrow$

$3\sqrt[3]{2}$

8

27 ←

~~64~~

~~125~~

ex: Divide, if possible. Then simplify.

c) $\dfrac{\sqrt[5]{640}}{\sqrt[5]{-4}}$

32 ✓

243

$-\sqrt[5]{160}$

$-\sqrt[5]{32}\cdot\sqrt[5]{5}$

$-2\sqrt[5]{5}$

ex: Perform the indicated operation. Give your answer in simplest form.

a) $\sqrt{2}\left(\sqrt{50}+7\right)$

$\sqrt{100}+7\sqrt{2}$

$10+7\sqrt{2}$

ex: Perform the indicated operation. Give your answer in simplest form.

b) $\left(1-\sqrt[5]{2}\right)\left(2+\sqrt[5]{2}\right)$

$2+\underline{\sqrt[5]{2}}-\underline{2\sqrt[5]{2}}-\sqrt[5]{4}$

$2-\sqrt[5]{2}-\sqrt[5]{4}$

ex: Perform the indicated operation. Give your answer in simplest form.

c) $\left(3+\sqrt[3]{4}\right)^2$

$$\left(3+\sqrt[3]{4}\right)\left(3+\sqrt[3]{4}\right)$$

$$9+\underline{3\sqrt[3]{4}}+\underline{3\sqrt[3]{4}}+\sqrt[3]{16}$$

$$\sqrt[3]{8}\cdot\sqrt[3]{2}$$

$$9+6\sqrt[3]{4}+2\sqrt[3]{2}$$

Rationalizing Denominators

A denominator is rationalized when no radicals exist in the denominator.

ex: Rationalize the denominator.

a) $\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

ex: Rationalize the denominator.

b) $\frac{2}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{9}}{\sqrt[3]{9}} = \frac{2\sqrt[3]{9}}{3}$

$\sqrt[3]{27}$

ex: Rationalize the denominator.

c) $\frac{1}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{8}}{\sqrt[4]{8}} = \frac{\sqrt[4]{8}}{2}$

$\sqrt[4]{16}$

2

ex: Rationalize the denominator.

d) $\frac{5}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}}$

$\frac{5\sqrt[3]{3}}{3}$

$\frac{\cancel{10}^{2}\sqrt[3]{3}}{\cancel{5}}$

ex: Rationalize the denominator.

e) $\dfrac{7}{\sqrt[6]{8}}$

$$\frac{7}{\sqrt[6]{2\cdot 2\cdot 2}} \cdot \frac{\sqrt[6]{2\cdot 2\cdot 2}}{\sqrt[6]{2\cdot 2\cdot 2}}$$

$$\frac{7\sqrt[6]{8}}{2}$$

$$\frac{\sqrt[6]{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2}}{2}$$

$$\sqrt[3]{27} = \sqrt[\cancel{3}]{3^{\cancel{3}}} = 3$$

$$\sqrt[4]{625} = \sqrt[\cancel{4}]{5^{\cancel{4}}} = 5$$

$$\sqrt{\frac{18}{2}}$$

$$\sqrt{9} = 3$$

ex: Rationalize the denominator.

f) $\frac{15}{\sqrt[5]{25}} \cdot \frac{\sqrt[5]{125}}{\sqrt[5]{125}} = \frac{15\sqrt[5]{125}}{5}$

$\sqrt[5]{5^5} = 3\sqrt[5]{125}$

5

ex: Rationalize the denominator.

g) $\dfrac{-8}{\sqrt[10]{16}}$ $\dfrac{\sqrt[10]{64}}{\sqrt[10]{64}} = \dfrac{-8\sqrt[10]{64}}{2}$

have 4 2's

need 6 2's

$-4\sqrt[10]{64}$

$\sqrt[\cancel{10}]{2^{\cancel{10}}}$

$2^6 = 64$

ex: Perform the indicated operation. Rationalize the denominator when necessary.

a) $\dfrac{\sqrt{2}}{\sqrt{6}}$

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{\sqrt{3}}{3}$$

ex: Perform the indicated operation. Rationalize the denominator when necessary.

b) $\dfrac{\sqrt[3]{12}}{\sqrt[3]{10}}$

$$\frac{\sqrt[3]{6}}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{25}}{\sqrt[3]{25}} = \frac{\sqrt[3]{150}}{5}$$

$\sqrt[3]{125} = 5$