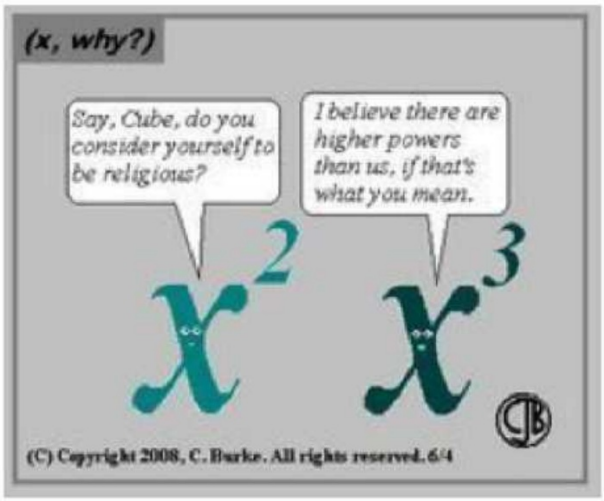
Properties of Exponents
Introduction to Polynomial Functions
Polynomial Operations - Addition and Subtraction



HW:

REVIEW: Exponent Properties

Property Name	Definition
Product of Powers	$a^m \cdot a^n = a^{m+n}$
Power of a Power	$(a^m)^n = a^{m \cdot n}$
Power of a Product	$(ab)^m = a^m \cdot b^m$
Negative Exponent	$a^{-m} = \frac{1}{a^m}$
Zero Exponent	$a^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

ex: Simplify.

a) $x^2 \cdot x^7 = x^9$

b) $\left(5x^7\right)^2 = 5^2 x^{14}$

$= 25x^{14}$

ex: Simplify. (do not use negative exponents)

c) $\frac{x^2}{x^7} = x^{2-7} = x^{-5} = \frac{1}{x^5}$

$$\frac{\cancel{x}\cancel{x}}{\cancel{x}\cancel{x}xxxxx}$$

d) $\frac{3x^{-2}}{x^7}$ $\quad 3x^{-9} = \frac{3}{x^9}$ $\qquad \frac{3}{x^2 \cdot x^7} = \frac{3}{x^9}$

$-2-7$

ex: Simplify.

e) $\left(2x^5\right)\left(5x^{-4}\right)^3\left(3x^0\right)$

$\left(2x^5\right)\left(125x^{-12}\right)\left(3\right)$

$750x^{-7}$

$\frac{750}{x^7}$

ex: Simplify.

f) $\dfrac{3x^2y^2}{2\cancel{x^{-1}}\left(4x^2y\right)^3}$

$x^{-1} = \dfrac{1}{x}$

$\dfrac{xxx}{xxxxxx}$

$\dfrac{x^1 3 x^2 y^2}{2 \cdot 64 x^6 y^3}$

$\dfrac{3x^3y^2}{128x^6y^3} = \dfrac{3}{128x^3y}$

ex: Simplify.

g) $$\frac{\left(2x^3z^0\right)^3}{\left(x^3y^{-4}z^2\right)\left(x^4z^{-3}\right)}$$

Monomial - a number, a variable or a product of numbers and variables

$7x, \; x^3, \; xy^3, \; 5$

Polynomial - an expression involving one or more monomials

$5, \; x+3, \; x^2-x+7$

Characteristics of Polynomials

1. variables have whole exponents

$0, 1, 2, 3, \ldots$

2. real coefficients

$-6, \sqrt{5}, 100, \pi$

3. no division by variables

ex: Determine whether the expression represents a polynomial.

a) $2x^2 - 4x + \frac{1}{7}$

Yes

b) 0

Yes

ex: Determine whether the expression represents a polynomial.

c) $\frac{5}{x^2}$ no

d) $\frac{1x^2}{5}$ Yes

$\frac{1}{5}x^2$

Classifying Polynomials

1. Degree - largest exponent

	Degree	Type
	0	constant
$2x+1$	1	linear
x^2+4	2	Quadratic
x^3	3	Cubic
x^4	4	quartic
x^5	5	quintic
	≥ 6	n^{th} degree poly.

Classifying Polynomials

2. Number of terms

Number of Terms	Type
1	monomial
2	binomial
3	trinomial
≥4	polynomial

ex: State the degree and number of terms. Then classify.

biggest exponent

a) $4x - 27x^2 + 3$

$-27x^2 + 4x + 3$

Quadratic trinomial

degree: 2

terms: 3

b) $3x + 7$

degree: 1

terms: 2

Linear binomial

ex: State the degree and number of terms. Then classify.

c) $5x^6 + 2x^3 + 4x - 5$

degree: 6 6th degree polynomial

terms: 4

d) $5x^4$

degree: 4 Quartic monomial

terms: 1

Standard Form of a Polynomial - a polynomial is in standard form when the terms' exponents are in descending order.

ex: Write the polynomial in standard form.

$$1 + 2x - 3x^4$$

$$-3x^4 + 2x + 1$$

<u>Leading Coefficient</u> - the coefficient of the term that defines the degree

ex: Identify the leading coefficient. (put in standard form)

degree: 4

$$\frac{1}{5}x - 3x^4 + 10$$

$$-3x^4 + \frac{1}{3}x + 10$$

$$\boxed{-3}$$

ex: Consider the four polynomial functions.

$$a(x) = -5 \qquad b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3 \qquad d(x) = 2x - 1$$

Perform the indicated operation. Write the answer in standard form.

a) $a(x) + b(x)$

$-5 + 5x^4 + 2$

$5x^4 - 3$

ex: Consider the four polynomial functions.

$$a(x) = -5 \qquad b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3 \qquad d(x) = 2x - 1$$

Perform the indicated operation. Write the answer in standard form.

b) $b(x) - c(x)$

$(5x^4 + 2) - (5x^2 + 4x - 3)$

$5x^4 + 2 - 5x^2 - 4x + 3$

$5x^4 - 5x^2 - 4x + 5$

ex: Consider the four polynomial functions.

$$a(x) = -5 \qquad b(x) = 5x^4 + 2$$

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Perform the indicated operation. Write the answer in standard form.

c) $8c(x)$

$8(5x^2+4x-3)$

$40x^2+32x-24$

ex: Consider the four polynomial functions.

$$a(x) = -5 \qquad b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3 \qquad d(x) = 2x - 1$$

Perform the indicated operation. Write the answer in standard form.

d) $d(x) - 5b(x)$

$(2x-1) - 5(5x^4+2)$

$2x - 1 - 25x^4 - 10$

$-25x^4 + 2x - 11$