Algebra 2: Solving polynomial equations (continued)

HW: Day 8 WKST(#2-8even,7)
AND
odd questions on the review

Find all possible rational zeros.

1) $f(x) = \overset{q}{4}x^3 - x^2 - 4x + \overset{p}{2}$

p: $\pm 1, \pm 2$

q: $\pm 1, \pm 2, \pm 4$

$\frac{p}{q}$: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2$

Find all possible rational zeros.

2) $f(x) = 5x^3 - 14x^2 - 58x - 11$

Factorable: 2, 3, and 5

Not factorable: 1, 4, and 6
(list the rational zeros; find
one that 'works' and then
continue solving.

Find all possible rational zeros.

3) $f(x) = \overset{q}{x^5} - 12x^2 + 24x - \overset{p}{8}$

$p: \pm 1, \pm 2, \pm 4, \pm 8$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8$

Solve.

1) $x^3 + 5x^2 + 7x + 3 = 0$

$\frac{p}{q}: \pm 1, \pm 3$

$$\begin{array}{r|rrrr} -3 & 1 & 5 & 7 & 3 \\ & & -3 & -6 & -3 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$\boxed{-3, -1 \text{ (mult of 2)}}$

Solve.

2) $2x^3 + 3x^2 + x = 0$

$x(2x^2 + 3x + 1) = 0$

$x(2x + 1)(x + 1) = 0$

$\boxed{x = 0, -\frac{1}{2}, -1}$

Solve.

3) $4x^3 - x^2 - 4x + 1 = 0$

$x^2(4x-1) - 1(4x-1) = 0$

$(x+1)(x-1)(4x-1) = 0$

$\boxed{-1, 1, 1/4}$

Solve.

4) $\overset{a}{x^3} - 5x^2 - \overset{p}{29x - 15} = 0$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & -29 & -15 \\ & & 1 & -4 & \\ \hline & 1 & -4 & -33 & \end{array}$$

$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15$

$$\begin{array}{r|rrrr} -3 & 1 & -5 & -29 & -15 \\ & & -3 & 24 & 15 \\ \hline & 1 & -8 & -5 & 0 \end{array}$$

$x^2 - 8x - 5 = 0$

$a = 1, b = -8 \quad c = -5$

$$\frac{8 \pm \sqrt{(64 - 4(1)(-5)}}{2(1)}$$

$$\frac{8 \pm \sqrt{84}}{2} = \frac{8 \pm 2\sqrt{21}}{2} \quad \frac{8}{2} \pm \frac{2\sqrt{21}}{2}$$

$\boxed{-3,\ 4 \pm \sqrt{21}}$

Solve.

5) $x^3 + 5x^2 - 2x - 10 = 0$

$x^2(x+5) - 2(x+5) = 0$

$(x^2-2)(x+5) = 0$

$\boxed{x = \pm\sqrt{2}, -5}$

Solve.

6) $2x^3 + 5x^2 + x - 2 = 0$

$\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}$

$$\begin{array}{r|rrrr} -2 & 2 & 5 & 1 & -2 \\ & & -4 & -2 & 2 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$

$2x^2 + x - 1 = 0$

$(2x-1)(x+1) = 0$

$\boxed{-2, \frac{1}{2}, -1}$

Complex Conjugate and Irrational Conjugate Theorem

Imaginary and irrational roots always come in conjugate pairs

Write a polynomial function, f(x), in standard form with integral coefficients and the given roots.

1) $-2, \sqrt{6}, 0$, $-\sqrt{6}$

$(x+2)\,x\,(x-\sqrt{6})(x+\sqrt{6})$

$x^2 + \cancel{\sqrt{6}x} - \cancel{\sqrt{6}x} - 6$

$(x+2)\,x\,(x^2-6)$

$f(x) = (x^2+2x)(x^2-6)$

$f(x) = x^4 + 2x^3 - 6x^2 - 12x$

Write a polynomial function, f(x), in standard form with integral coefficients and the given roots.

2) $4, 5i, -5i$

$(x-4)\underbrace{(x-5i)(x+5i)}$

$(x-4)(x^2-25i^2)$

$(x-4)(x^2+25) = \boxed{f(x) = x^3-4x^2+25x-100}$

$(x^2+25)(x-4)=0$

$x^2(x-4)+25(x-4)$

Write a polynomial function, f(x), in standard form with integral coefficients and the given roots.

3) $-\sqrt{3}$, multiplicity of 2

$\sqrt{3}$

$(x+\sqrt{3})^2(x-\sqrt{3})^2$

$-\sqrt{3}$

$\sqrt{3}$

$$\underbrace{(x+\sqrt{3})(x-\sqrt{3})}\underbrace{(x+\sqrt{3})(x-\sqrt{3})}=f(x)$$

$$(x^2-3)(x^2-3)=f(x)$$

$$f(x)=x^4-6x^2+9$$

Review: Divide using long division.

$$(x^2 - 6x + 11) \div (x - 4)$$

Simplify.

$$(3x^5y^{-3})^2(6x^{-2}y)^{-1}$$

Is (x - 2) a factor of $x^3 - 6x^2 - 7x + 1$ *?*

Factor completely.

$$27x^4 + 64x$$