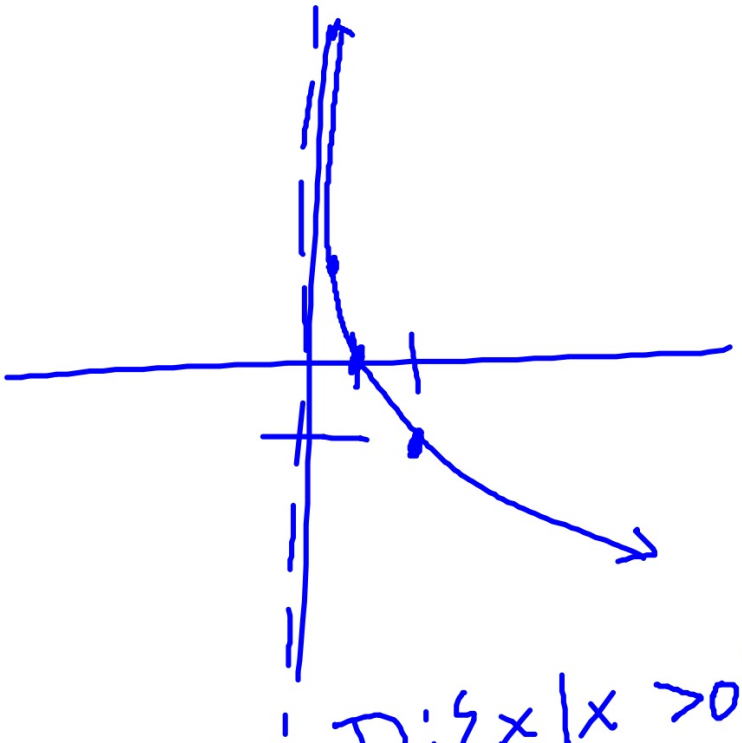
$$y = \log_{1/2} x$$

$$\frac{1}{2} \quad \frac{1}{2}$$

$$\left(\frac{1}{2}\right)^y = x$$

x	y
2	-1
1	0
1/2	1



$D: \{x \mid x > 0\}$

$R: \{y \mid y \in R\}$

7.) $2^r \cdot 2^{r+3} = 2^r$

$2^{r+r+3} = 2^r$

$2r+3 = r$

$r = -3$

$\cancel{2^r} \cdot 2^{r+3} = \cancel{2^r}$

$2^{r+3} = 1$

$2^{r+3} = 2^0$

$r+3 = 0$

2.) $5^{2-3k} = \left(\frac{1}{25}\right)^1$

$5^{2-3k} = 5^{-2}$

$2 - 3k = -2$

$-3k = -4$

$k = 4/3$

5.) $625^{2n+2} = 25^{-n}$

$$5^{4(2n+2)} = 5^{-2n}$$

$$8n + 8 = -2n$$

$$\frac{8}{-10} = \frac{-10n}{-10}$$

$$-\frac{8}{10} = n$$

$$-4/5 = n$$

8.) $\left(\frac{1}{25}\right)^{-p} = 625^{3p}$

$5^{2p} = 5^{12p}$

$2p = 12p$

$0 = 10p$

$0 = p$

$$y = \log(x-2) + 4$$

$$y - 4 = \log(x-2)$$

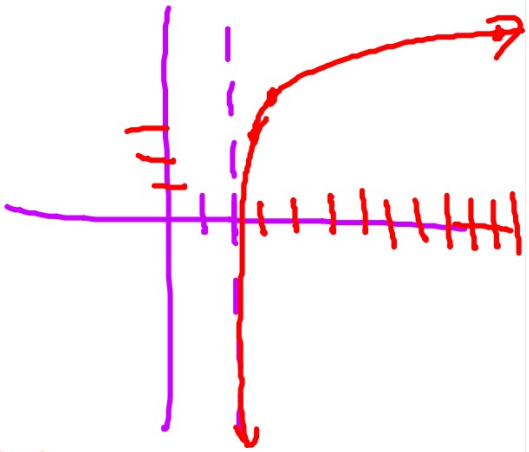
$$10^{\quad} \qquad 10^{\quad}$$

$$10^{y-4} = x-2$$

$$10^{y-4} + 2 = x$$

LA $x=2$

x	y
$10^{-1}+2=2\frac{1}{10}$	3
$10^{0}+2=3$	4
$10^{1}+2=12$	5



$$y = \ln x - 5$$

$+5 \qquad +5$

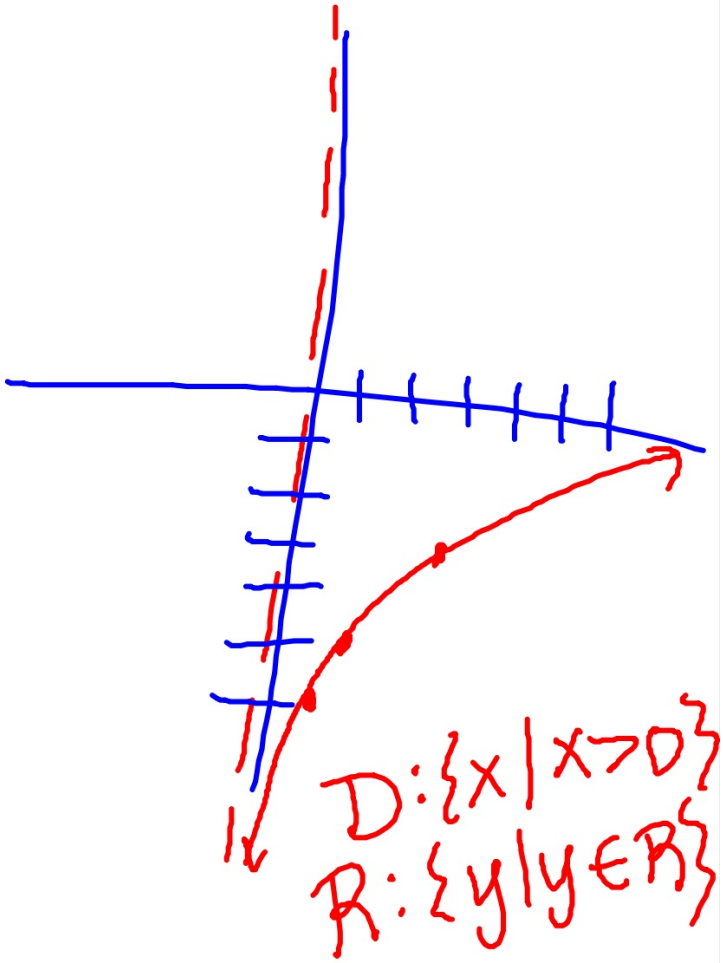
$$y + 5 = \ln x$$

$$e^{y+5} = e^{\ln x}$$

$$e^{y+5} = x$$

VA
$x = 0$

x	y
.37	−6
1	−5
2.72	−4



A2 Solving Exponential & Logarithmic Equations Day 2

Logic: another thing that penguins aren't very good at.

HW:

2 Types of Exponential Equations:

1. $a^x = b$, where a and b are integral powers of the same number

$$\text{ex: } 27^x = 9$$

2. $a^x = b$, where a and b are NOT integral powers of the same number

$$\text{ex: } 3^x = 5$$

REVIEW:

ex: Solve.

$$27^{3x-7} = 81^{12-3x}$$

$$3^{3(3x-7)} = 3^{4(12-3x)}$$

$$9x - 21 = 48 - 12x$$

$$+12x \qquad +12x$$

$$21x - 21 = 48$$

$$21x = 69$$

$$x = \frac{69}{21} = \frac{23}{7}$$

Type 2

ex: Solve.

a) $3^x = 5$

$\log 3^x = \log 5$

$\frac{x \cancel{\log 3}}{\cancel{\log 3}} = \frac{\log 5}{\log 3}$

$x = \frac{\log 5}{\log 3}$

$\boxed{1.465}$

ex: Solve.

b) $e^{x+1} = 10$

$\ln e = 1$

$\ln 1 = 0$

$\ln e^{x+1} = \ln 10$

$(x+1) = \ln 10$

$x = -1 + \ln 10$

$x = 1.303$

Check

$e^{1.303+1} \stackrel{?}{=} 10$

$e^{2.303}$

ex: Solve.

c) $3 \cdot \boxed{4^{x-7}} + 6 = 54$

$$\frac{\cancel{3} \cdot 4^{x-7}}{\cancel{3}} = \frac{48}{3}$$

$$\log 4^{x-7} = \log 16$$

$$\frac{(x-7)\cancel{\log 4}}{\cancel{\log 4}} = \frac{\log 16}{\log 4}$$

$$x - 7 = 2$$

$$x = 9$$

$$4^{x-7} = 16$$

$$4^{x-7} = 4^2$$

$$x - 7 = 2$$

ex: Solve.

d) $2 \cdot 10^{x-3} - 3 = 37$

$\log 10^{x-3} = \log 20$

$(x-3)\cancel{\log 10} = \log 20$

$x - 3 = \log 20$

$x - 3 = 1.301$

$x = 4.301$

ex: Solve.

e) $2 - 5^{x-2} = 3$

$-5^{x-2} = 1$

$\log 5^{x-2} = \log -1$

$(x-2)\log 5 = \cancel{\log(-1)}$

$\emptyset$

REVIEW: Evaluate.

a) $\log_3\left(\frac{1}{9}\right)$

b) $\frac{\log 36}{\log 6}$

c) $\log_8(-8)$

d) $\log 0$

Domain of Logarithmic Functions

$$y = \log_b(f(x))$$

Domain: $f(x) > 0$

ex: State the domain in set notation.

a) $y = \log_2(x-1)$

$x - 1 > 0$

$\{x \mid x > 1\}$

ex: State the domain in set notation.

b) $y = 3 - \ln(-x)$

$-x > 0$

$\{x \mid x < 0\}$

c) $y = \log_9(x^2 + 9)$

$x^2 + 9 > 0$

$x^2 > -9$ True at all times

$\{x \mid x \in R\}$

Solving Logarithmic Equations

Two Types:

1. 1 Logarithm

$$\text{ex: } 2 - \log_2(x+1) = 4$$

2. More than 1 Logarithm

$$\text{ex: } \log_3(x^2 - 3) = \log_3 2 + \log_3 x$$

When solving type 1 or 2, rewrite the equation with ONE TERM on each side of the equation.

ex: Solve.

a) $\log_{36} x + \frac{1}{2} = 2$

$$\cancel{36}^{\log_{36} x} = 36^{\frac{3}{2}}$$

$$x = 36^{3/2}$$

$$\left(\sqrt{36}\right)^3 = 6^3 = 216$$

ex: Solve.

b) $\log_{12}\left(x^2-4\right)=\log_{12}\left(3x\right)$

$\cancel{12}$ $\cancel{12}$

$x^2-4=3x$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x=4, \cancel{-1}$

ex: Solve.

c) $\log_{31}(4x^2-5)-\log_{31}(2x^2-1)=0$

$$\cancel{31}^{\log_{31}} \frac{4x-5}{2x-1} = 31^0$$

$$\frac{4x-5}{2x-1} = 1$$

$$4x-5=2x-1$$

$$2x=4$$

$$x=2$$

ex: Solve.

condense

d) $\log_2(4x-5)-\log_2(2x-1)=3$

$$\cancel{2}^{\cancel{\log_2}\frac{4x-5}{2x-1}}=\cancel{2}^{3}$$

$$\frac{4x-5}{\cancel{2x-1}}=8(2x-1)$$

$$4x-5=16x-8$$

$$3=12x$$

$$x=\frac{1}{4}$$

$\emptyset$

ex: Solve.

e) $\log 18 - \log 3x = \log 2$

$$\cancel{10}^{\log}\frac{18}{3x} = \cancel{10}^{\log} 2$$

$$\frac{18}{\cancel{3x}} = 2 \cdot 3x$$

$$18 = 6x$$

$$3 = x$$

ex: Solve.

f) $\frac{\ln(-x)}{2} + \cancel{4} = 5$

$\cancel{-4} \quad -4$

$\frac{\cancel{2} \cdot \ln(-x)}{\cancel{2}} = 1 \cdot 2$

$\cancel{e}^{\ln(-x)} = e^{2}$

$-x = e^{2} \qquad x = -e^{2} = -7.389$

ex: Solve.

h) $\log_5 \sqrt{x-2} = 1$

$5^{\log_5 \sqrt{x-2}} = 5^1$

$\left(\sqrt{x-2}\right)^2 = (5)^2$

$x-2 = 25$

$x = 27$

i.) $\log_2 x + \log_2 (x+3) = 2$

$\log_2 x(x+3) = 2$

$2^{\log_2 (x^2+3x)} = 2^2$

$x^2 + 3x = 4$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x = \cancel{-4}, 1$