2.) $\dfrac{1 \cdot t}{8} + \dfrac{2 \cdot 8}{t} - \dfrac{17}{8t} = 0$

$$\dfrac{t + 16 - 17}{8t} = 0$$

$$\dfrac{t - 1}{8t} = 0$$

$t - 1 = 0$

$t = 1$

$$19.) \quad 2x \cdot \frac{4}{1} + \overbrace{\frac{1 \cdot 2}{\boxed{x}}} - \frac{10}{2x} = 0$$

$$\frac{8x + 2 - 10}{2x} = 0$$

$$\frac{4x + 1 - 5}{x}$$

$$\frac{8x - 8}{2x} = 0$$

$$\frac{8(x-1)}{2x} = 0$$

$$8(x-1) = 0$$
$$x = 1$$

$$1 \qquad \cancel{x} \qquad 2 \cdot \cancel{x}$$

$$1 \cdot x \cdot 2$$

$$2x$$

25.) $\dfrac{4x \cdot 2}{x-3} + \dfrac{x(x-3)}{2} - \dfrac{12 \cdot 2}{x-3} = 0$

$$\dfrac{8x + x^2 - 3x - 24}{2(x-3)} = 0$$

$$\dfrac{x^2 + 5x - 24}{2(x-3)} = 0$$

$$\dfrac{(x+8)(x-3)}{2(x-3)} = 0$$

$\boxed{-8}$

43.) $\dfrac{1(a+1)}{a-1} + \dfrac{4(a-1)}{a+1} - \dfrac{7}{(a+1)(a-1)} = 0$

$$\dfrac{a+1+4a-4-7}{(a+1)(a-1)} = 0$$

$$\dfrac{5a-10}{(a+1)(a-1)} = 0$$

$$5a - 10 = 0$$

$$\boxed{a = 2}$$

9.) $\dfrac{2 \cdot x}{x-3} + \dfrac{x(x-3)}{2} - \dfrac{6x}{2(x-3)} = 0$

$\dfrac{2x + x^2 - 3x - 6x}{2(x-3)} = 0$

$\dfrac{x^2 - 7x}{2(x-3)} = 0$

$\dfrac{x(x-7)}{2(x-3)} = 0$

$x \neq 3$

$x = 0,7$

Graphs of Rational Functions

When sketching rational functions, find:

x-intercepts
y-intercept
Domain
Asymptotes
Holes

Finding x-intercepts: Simplify the function; set the numerator equal to zero.

a) Find the x-intercept(s), if any.

$$y = \frac{7x}{x+5}$$

$7x = 0$

$x = 0$

$(0, 0)$

b) Find the x-intercept(s), if any.

$$g(x) = \frac{5}{x}$$

$5 \neq 0$

no x-int.

if the numerator doesn't have a variable, there is no x-int.

c) *Find the x-intercept(s), if any.*

$$y = \frac{x^2 + 3x}{x^2 - 16}$$

$$\frac{0}{-16} = 0$$

$$y = \frac{x(x+3)}{(x+4)(x-4)}$$

$$x(x+3) = 0$$

$$x = 0 \qquad x + 3 = 0$$

$$0, \ -3$$

$$(0, 0)$$
$$(-3, 0)$$

d) *Find the x-intercept(s), if any.*

$$f(x) = \frac{x^2 - x - 12}{x^2 - 2x - 8}$$

$$f(x) = \frac{(x-4)(x+3)}{(x-4)(x+2)}$$

$$f(x) = \frac{x+3}{x+2}$$

$$x+3 = 0$$
$$x = -3$$

$$(-3, 0)$$

Finding the y-intercept: Plug in x = 0 and solve for y.

a) Find the y-intercept, if any.

$$y = \frac{7x}{x+5}$$

Any function will have at most 1 y-intercept.

$$y = \frac{0}{5} = 0$$

$$(0,0)$$

b) Find the y-intercept, if any.

$$g(x) = \frac{5}{x}$$

$$g(0) = \frac{5}{0} \nearrow \text{undefined}$$

no y-int.

c) *Find the y-intercept, if any.*
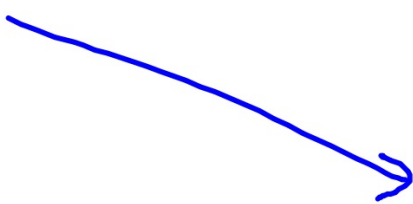
$$y = \frac{x^2 + 3x}{x^2 - 16}$$

$$y = \frac{0 + 0}{0 - 16} \qquad (0, 0)$$

$$= \frac{0}{-16}$$

$$= 0$$

d) *Find the y-intercept, if any.*

$$f(x) = \frac{x^2 - x - 12}{x^2 - 2x - 8}$$

$$\frac{0 - 0 - 12}{0 - 0 - 8}$$

$$\frac{3}{2}$$

$$\left(0, \frac{3}{2}\right)$$

Finding the domain: Set the denominator equal to zero. Solve for x.

a) Find the domain.

$$y = \frac{7x}{x+5}$$

$$x + 5 = 0$$
$$x = -5$$
$$\{x \mid x \neq -5\}$$

b) *Find the domain.*

$$g(x) = \frac{5}{x}$$

$$x = 0$$

$$\{x \mid x \neq 0\}$$

c) *Find the domain.*

$$y = \frac{x^2 + 3x}{x^2 - 16}$$

$$x^2 - 16 = 0$$
$$(x+4)(x-4) = 0$$
$$x = \pm 4$$
$$\{x \mid x \neq \pm 4\}$$

d) Find the domain.

$$f(x) = \frac{x^2 - x - 12}{x^2 - 2x - 8}$$

$$\hookrightarrow x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

$$x \neq 4, -2$$

$$\{x \mid x \neq 4, -2\}$$

Finding Horizontal Asymptotes

To find the horizontal asymptote compare the degree of the numerator and denominator. Three cases arise:

Case	Degree		Degree	Asymptote
1	Numerator	<	Denominator	$y = 0$
2	Numerator	>	Denominator	none
3	Numerator	=	Denominator	$y = \dfrac{a}{b}$

Rational functions can have at most ONE HA

Rembering Horizontal Asymptotes

BOBO   BOTN   EATSDC

Bigger	Bigger	Exponents
On	On	Are
Bottom	Top	The
0 (zero)	None	Same
		Divide
		Coefficients

$$y = 0$$

NO
HA

$$y = \frac{a}{b}$$

Horizontal Asymptotes

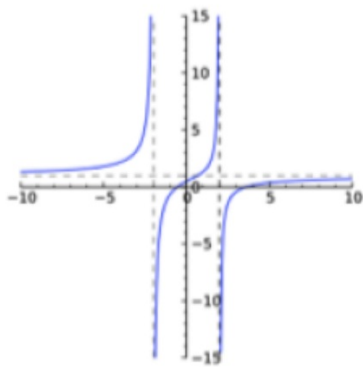ex: Find the HA, if any.

a) $y = \dfrac{16x + 1}{4x^2 - 2}$ HA: $y = 0$ (bobo)

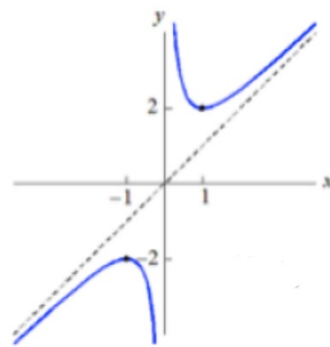b) $y = \dfrac{16x^2 + 1}{4x^2 - 2}$ HA: $y = \dfrac{16}{4}$ (eats dc)
$$y = 4$$

c) $y = \dfrac{16x^2 + 1}{4x - 2}$ no HA (both)

3 examples of graphs: BOBO, BOTN, EATS DC

BOBO

BOTN

EATS DC

$y = 2$

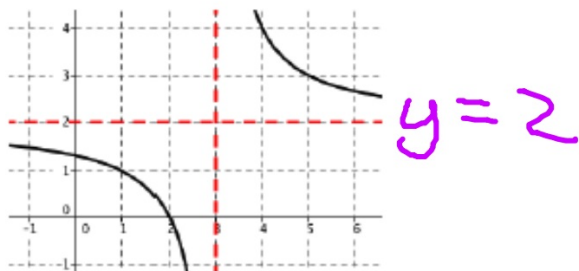*Finding Vertical asymptotes*

To find vertical asymptotes:

To find vertical asymptotes:

       1. Simplify.

       2. Set the simplified denominator = 0.

Rational functions can have more than one VA

Find the vertical asymptote(s), if any.

a) $y = \dfrac{x}{3x^2 - 2x - 8}$

$y = \dfrac{x}{(3x+4)(x-2)}$

$(3x+4)(x-2) = 0$

$x = -\dfrac{4}{3} \quad x = 2 \quad VA$

Find the vertical asymptote(s), if any.

b) $y = \dfrac{x+1}{x^2 + 16x + 15}$

$$y = \frac{\cancel{x+1}}{(x+15)\cancel{(x+1)}}$$

$$y = \frac{1}{x+15}$$

$$x + 15 = 0$$

$$\boxed{x = -15}$$

Find the vertical asymptote(s), if any.

c) $y = \dfrac{x^2}{x^2 + 9}$

$$x^2 + 9 = 0$$
$$x^2 = -9$$

imaginary

No VA

Find the vertical asymptote(s), if any.

d) $y = \dfrac{6x}{x-7}$

$$x - 7 = 0$$
$$x = 7$$

Finding Holes

To find holes:

      1. Factor completely.

      2. If the numerator and denominator share a common factor a hole exists.

      3. The hole exists at the zero of the common factor.

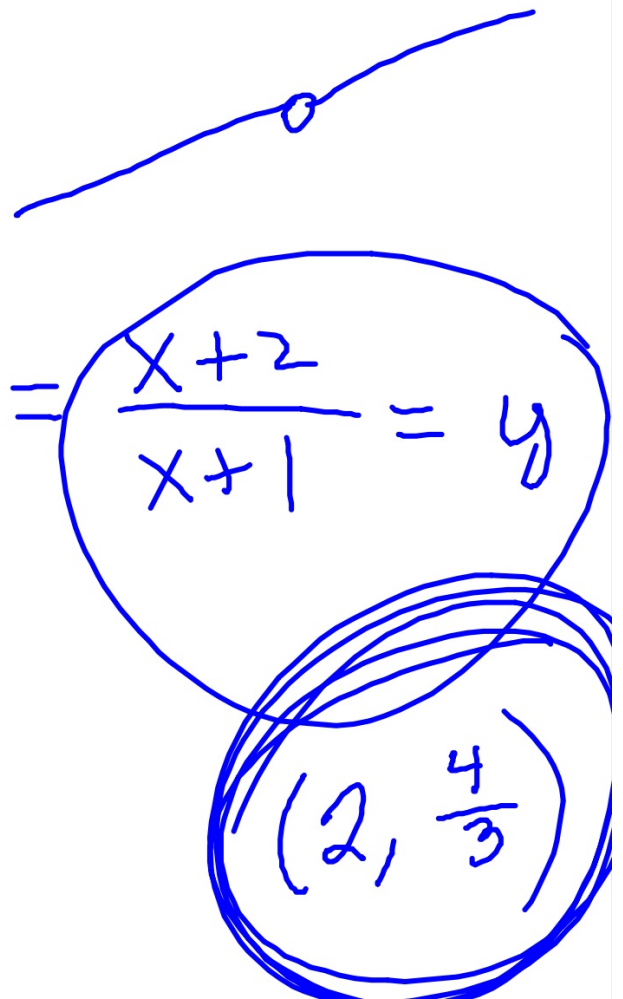   4. To find the y-value, plug in x into the SIMPLIFIED version.

Rational functions can have more than one hole

Find all holes, if any.

a) $\quad y = \dfrac{x^2 - 4}{x^2 - x - 2}$

$$y = \frac{(x-2)(x+2)}{(x-2)(x+1)} = \frac{x+2}{x+1} = y$$

$$x - 2 = 0$$
$$x = 2$$
$$(hole)$$

$$\left(2, \frac{4}{3}\right)$$

Find all holes, if any.

b) $y = \dfrac{x+5}{5x^2 - 3x - 2}$

Find all holes, if any.

c) $y = \dfrac{5x^2 - 9x - 18}{x^2 - 3x}$

Find all holes, if any.

d) $y = \dfrac{x^3 + x^2 + 3x + 3}{x^2 - 1}$

Review

a) *State the end behavior.* $y = 5x - 3x^3 + 2x^4$

b) *State the multiplicity of the zeros.*

$$y = (x+2)^2(x-2)$$

c) *Perform the indicated operation and simplify.*

$$\frac{5}{x+1} - \frac{3}{x-4}$$

d) Solve.

$$\frac{1}{x} + \frac{3}{x-1} = 0$$