

Simplifying radicals

Solving by quadratics by factoring

Complex numbers

Solving by square root method

Quadratic formula and the discriminant

Quadratic word problems

Solving Quadratics Using the Quadratic Formula

Let $a, b, c \in \mathbb{R}$ such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are:

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Use the Quadratic Formula to solve a quadratic equation when...

*a quadratic equation is NOT factorable
or we cannot use square root method*

ex: Solve.

a) $x^2 + x - 4 = 0$

$$\begin{aligned} a &= 1 \\ b &= 1 \\ c &= -4 \end{aligned}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 + 16}}{2}$$

$$\boxed{x = \frac{-1 \pm \sqrt{17}}{2}}$$

$$b.) \quad x^2 + 2x + 5 = 0 \quad \sqrt{-1} = i$$

$$\begin{aligned} a &= 1 \\ b &= 2 \\ c &= 5 \end{aligned}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm i\sqrt{16}}{2}$$

$$x = \frac{-2 \pm 4i}{2} = \frac{-2}{2} \pm \frac{4i}{2} = \boxed{-1 \pm 2i}$$

$$c.) \quad \begin{array}{ccc} 2x^2 & +2x & = 5-2x \\ & +2x & +2x \end{array} \quad \begin{array}{l} \sqrt{56} = \sqrt{4 \cdot 14} \\ = 2\sqrt{14} \end{array}$$

$$2x^2 + 4x - 5 = 0$$

$$a=2 \quad x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-5)}}{2(2)}$$

$$b=4$$

$$c=-5$$

$$x = \frac{-4 \pm \sqrt{16+40}}{4} = \frac{-4 \pm \sqrt{56}}{4}$$

$$x = \frac{-4 \pm 2\sqrt{14}}{4} = \frac{-4}{4} \pm \frac{2\sqrt{14}}{4} = \left(-1 \pm \frac{\sqrt{14}}{2} \right)$$

$$d.) \quad 3x^2 + 4x + 2 = 0$$

$$\sqrt{-8} = i\sqrt{4 \cdot 2}$$

$$a = 3$$

$$b = 4$$

$$c = 2$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(2)}}{2(3)}$$

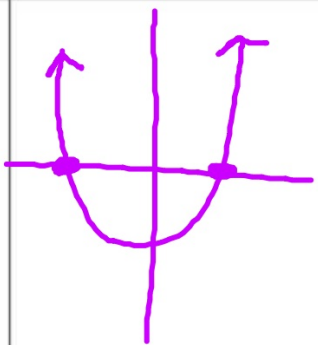
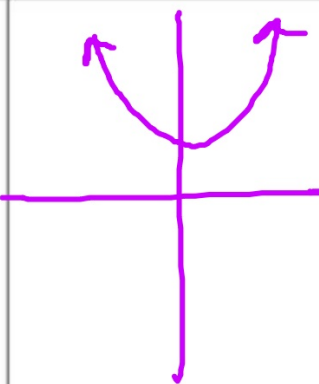
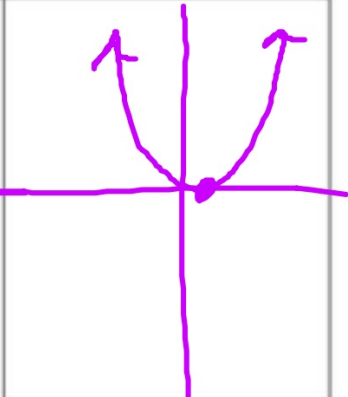
$$x = \frac{-4 \pm \sqrt{-8}}{6}$$

$$x = \frac{-4 \pm 2i\sqrt{2}}{6} = \frac{-4}{6} \pm \frac{2i\sqrt{2}}{6} = \left(\frac{-2}{3} \pm \frac{i\sqrt{2}}{3} \right)$$

The Discriminant:

- In the quadratic formula, the expression $b^2 - 4ac$ is called the discriminant.
- The discriminant is used to determine the *types of solutions for the equation*.

Using The Discriminant:

Value of discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac < 0$	$b^2 - 4ac = 0$
Number of solutions	2	2	1 repeated
Type of solutions	real	imaginary	real
Graph of $y = ax^2 + bx + c$			

ex: Find the discriminant and give the number and type of solutions of the equation.

a) $x^2 - 8x + 13 = -4$

$$x^2 - 8x + 17 = 0$$

$$a = 1$$

$$b = -8$$

$$c = 17$$

$$b^2 - 4ac$$

$$(-8)^2 - 4(1)(17)$$

$$64 - 68$$

$$-4$$

2 imaginary solutions

b) $x^2 - 8x + 16 = 0$

$$a = 1$$

$$b = -8$$

$$c = 16$$

$$b^2 - 4ac$$

$$(-8)^2 - 4(1)(16)$$

$$64 - 64$$

$$0$$

1 real solution

$$c) 8x^2 - 2x + 1 = x^2 + 6$$

$$\begin{array}{cc} -x^2 & -6 - x^2 - 6 \end{array}$$

$$7x^2 - 2x - 5 = 0$$

$$a = 7$$

$$b = -2$$

$$c = -5$$

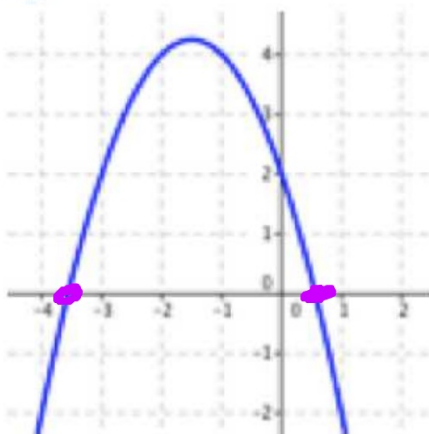
$$(-2)^2 - 4(7)(-5)$$

$$4 + 140$$

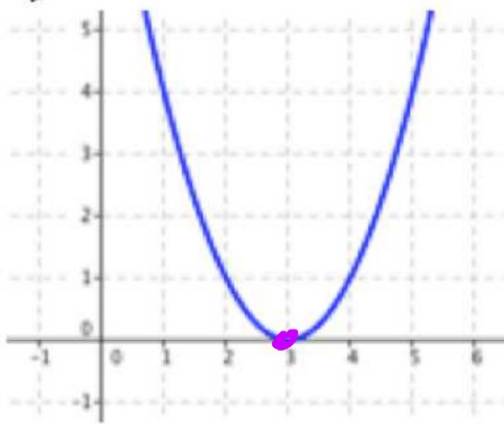
144
2 real

ex: The graph of $y = ax^2 + bx + c$ and the solutions of $ax^2 + bx + c = 0$ are given. Determine if the discriminant is positive, negative, or zero. Explain your reasoning.

a)



b)



zero

c) $x = 2 \pm 3i$

↑ imaginary

negative

Solve.

$$x^2 - 3x - 18 = 0$$
$$(x-6)(x+3) = 0$$
$$x = 6, -3$$

$$3(x+2)^2 - 1 = 20$$
$$\sqrt{(x+2)^2} = \sqrt{7}$$
$$x+2 = \pm\sqrt{7}$$
$$x = -2 \pm \sqrt{7}$$

Simplify

$$3\sqrt{84} = 3\sqrt{4 \cdot 21}$$
$$= 6\sqrt{21}$$

$$(2+i)(3-4i)$$
$$6 - 8i + 3i - 4i^2$$
$$6 - 5i + 4 = 10 - 5i$$