

$$\frac{\log_8 x}{2} + \frac{\log_8 y}{2} + \frac{\log_8 z}{2}$$

$$\frac{1}{2} \left(\log_8 x + \log_8 y + \frac{1}{2} \log_8 z \right)$$

$$\frac{1}{2} \log_8 (xyz) = \log_8 \sqrt{xyz}$$

$$3.) e^{\ln 2} = 2$$

$$e^{\ln x} = x$$

$$(6.) \log_3 \frac{(x+5)^2}{81}$$
$$\log_3 (x+5)^2 - \log_3 81$$
$$2\log_3 (x+5) - 4$$

$$7.) \ln \sqrt{e^3 x^4 y}$$

$$\ln(e^3 x^4 y)^{\frac{1}{2}}$$

$$\frac{1}{2} \ln(e^3 x^4 y)$$

$$\frac{1}{2} \ln e^3 + \frac{1}{2} \ln x^4 + \frac{1}{2} \ln y$$

$$\frac{3}{2} + 2 \ln x + \frac{1}{2} \ln y$$

$$\log 1 = 0$$

$$\begin{aligned}\ln e &= 1 \\ \ln 1 &= 0\end{aligned}$$

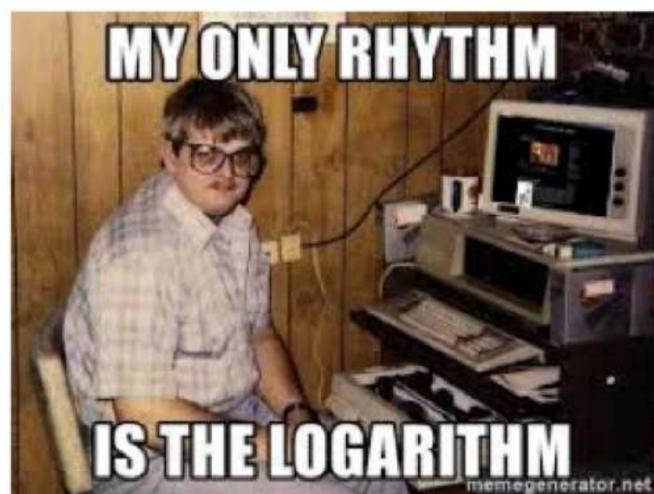
$$\begin{aligned}\ln e &= 1 \\ \log_e e &= 1 \\ e^{\log_e} &= e\end{aligned}$$

$$2.) \log_7 7^5$$

$$5 \log_7 7$$

5

A2 - Properties of Logarithms Day 2



REVIEW - Exponent Properties

$$b^m \cdot b^n =$$

$$\frac{b^m}{b^n} =$$

$$(b^m)^n =$$

Logarithm Properties

Let b , m , and n be positive numbers such that $b \neq 1$.

Product Property $\log_b mn =$

Quotient Property $\log_b \frac{m}{n} =$

Power Property $\log_b m^n =$

Logarithm properties are used to EXPAND and CONDENSE logarithmic expressions.

ex: Expand.

a) $\log_3\left(\frac{abc}{9d}\right)$

$$\log_3 a + \log_3 b + \log_3 c - \log_3 9 - \log_3 d$$

$$\log_3 a + \log_3 b + \log_3 c - 2 - \log_3 d$$

ex: Expand.

b) $\log\left(\frac{100a^2}{b^3c}\right)$

$$\log 100 + \log a^2 - \log b^3 - \log c$$

$$2 + 2 \log a - 3 \log b - \log c$$

ex: Expand.

c) $\log_3(a + b^2)$

already
simplified

ex: Expand.

d) $\log_4\left(\frac{a+b}{a-b^2}\right)$

$$\log_4(a+b) - \log_4(a-b^2)$$

e) $\log_2(a^2 - b^2) = \log_2(a+b)(a-b)$

$$\log_2(a+b) + \log_2(a-b)$$

ex: Expand.

f) $\log_3(a-b)^7$

$$\log \sqrt[7]{a-b}$$

$$7 \log_3(a-b)$$

g) $\ln \left(\frac{y^3 + z}{x^3(a+1)^5} \right)^{1/2}$

$$\frac{1}{2} \ln \frac{y^3 + z}{x^3(a+1)^5} = \frac{1}{2} \ln(y^3 + z) - \frac{1}{2} \ln x^3 - \frac{1}{2} \ln(a+1)^5$$
$$\frac{1}{2} \ln(y^3 + z) - \frac{3}{2} \ln x - \frac{5}{2} \ln(a+1)$$

$$\log_{32} 8$$

$$32^{\square} = 8$$

$$2^{5\square} = 2^3$$

$$5x = 3$$

ex: Expand.

$$\text{h) } \log_2 \sqrt[3]{\frac{16a^5}{b^2 + c^2}} = \frac{1}{3} \log_2 \frac{16a^5}{b^2 + c^2}$$
$$= \frac{1}{3} \log_2 16 + \frac{1}{3} \log_2 a^5 - \frac{1}{3} \log_2 (b^2 + c^2)$$
$$= \frac{4}{3} + \frac{5}{3} \log_2 a - \frac{1}{3} \log_2 (b^2 + c^2)$$

$$\text{i) } \log_{32} \left(\frac{x^3 - y^3}{8} \right) = \log_{32} (x-y)(x^2 + xy + y^2) - \log_{32} 8$$
$$= \log_{32} (x-y) + \log_{32} (x^2 + xy + y^2) - \frac{3}{5}$$

ex: Condense.

a) $2 \log_5 a - 3 \log_5 b + 4 \log_5 (c+d)$

$$\log_5 a^2 - \log_5 b^3 + \log_5 (c+d)^4$$

$$\log_5 \left(\frac{a^2 (c+d)^4}{b^3} \right)$$

ex: Condense.

b) $\log x + \log y - \cancel{10} \log z^{10}$

$$\log\left(\frac{xy}{z^{10}}\right)$$

ex: Condense.

c) ~~$-3\log x - 4\log y - \frac{2}{3}\log z$~~

c) $\frac{1}{3}\log x - \frac{4}{3}\log y - \frac{5}{3}\log z$

$$\frac{1}{3}(\log x - 4\log y^4 - 5\log z^5)$$

$$\frac{1}{3}\log\left(\frac{x}{y^4z^5}\right) = \log\sqrt[3]{\frac{x}{y^4z^5}}$$

ex: Condense.

d) ~~$\frac{1}{3}\log_4 a + \frac{2}{3}\log_4 b - \frac{4}{3}\log_4 c$~~

$$\log(x^2 - 9) - \log(x+3)$$

$$\log \frac{x^2 - 9}{x+3}$$

$$\log\left(\frac{(x+3)(x-3)}{x+3}\right) = \log(x-3)$$

Change of Base Formula

$$\log_b x = \frac{\log_a x}{\log_a b}$$

a: any
valid
log base

Change of base formula is NOT used to expand or condense

$$\log_5 17 \quad \log_2 16$$



ex: Rewrite using common or natural logarithms.
Then evaluate on your calculator.

$$\text{a) } \log_2 11 = \frac{\log 11}{\log 2} = 3.459$$

$$= \frac{\ln 11}{\ln 2} =$$

$$\text{b) } \log_3 25 \quad 2.930$$

Rewrite the log using the change of base formula.

$$\log_7 40 = \frac{\log 40}{\log 7}$$

OR

$$\frac{\ln 40}{\ln 7}$$

$$\log_3 \frac{1}{81} = -4$$

$$3^{\square} = \frac{1}{81}$$

$$3^{\square} = 3^{-4}$$

$$\log_9 27 = \frac{3}{2}$$

$$9^{\square} = 27$$

$$3^{2\square} = 3^3$$

$$2x = 3$$

$$\ln e^4 = \log_e e^4 = 4$$