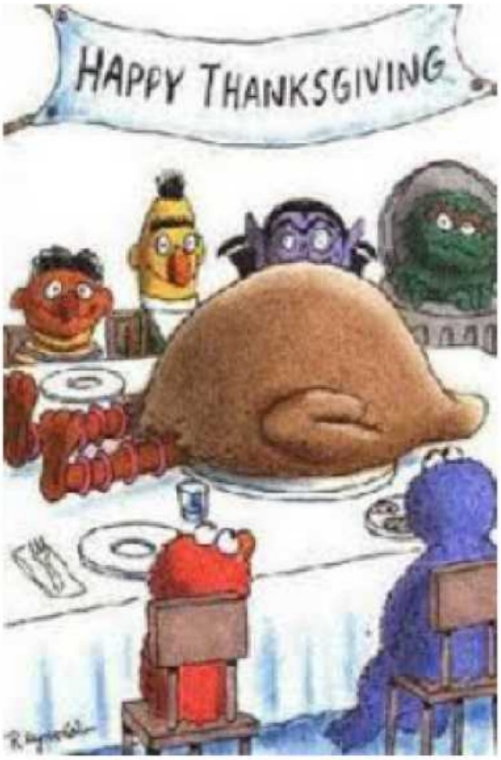
Polynomial Division



Polynomial Division Techniques

1. Dividing By A Monomial

2. Dividing By A Polynomial
   - Long Division
   - Synthetic Division

1. Dividing By A Monomial

ex: Divide.

$$\frac{7x^4 - 5x^2 + 14x}{21x^3}$$

$$\frac{7x^4}{21x^3} - \frac{5x^2}{21x^3} + \frac{14x}{21x^3}$$

$$\frac{x}{3} - \frac{5}{21x} + \frac{2}{3x^2}$$

$$\frac{6+12}{2}$$

$$\frac{6}{2} + \frac{12}{2}$$

2. Dividing By A Polynomial - Long Division

REVIEW: Divide.

$523 \div 4$

$$
\begin{array}{r}
130\frac{3}{4} \\
4\overline{)523} \\
-4\downarrow \\
\hline
12 \\
12 \\
\hline
03 \\
-0 \\
\hline
3
\end{array}
$$

$\div$

$\cdot$

$\times$

$-$

ex: Divide using long division.

a) $\dfrac{x^2+2x+7}{x-8}$

Standard form for numerator + denominator?

$$x + 10 + \frac{87}{x-8}$$

$$\begin{array}{r} x + 10 \\ x-8 \overline{\big) x^2 + 2x + 7} \\ + (-x^2 + 8x) \\ \hline 10x + 7 \\ + (-10x + 80) \\ \hline 87 \end{array}$$

$\dfrac{10x}{x}$

$10(x-8)$

$\div \dfrac{x^2}{x}$

$x \quad x(x-8)$

$$\begin{array}{r} 7 \\ 9 \overline{\big) 71} \\ -63 \\ \hline \end{array}$$

ex: Divide using long division.

b) $\dfrac{x^2+3x-40}{x+5}$

$$x - 2 + \frac{-30}{x+5}$$

$$\begin{array}{r|l} x+5 & x^2+3x-40 \\ & -(x^2+5x) \\ \hline & -2x-40 \\ & -(-2x-10) \\ \hline & -30 \end{array}$$

$$\frac{-2x}{x}$$

ex: Divide using long division.

c) $\dfrac{6x^3 - 11x^2 + 14x - 11}{2x - 1}$

ex: Divide using long division.

d) $\dfrac{5x^3 - 2x^2 + 1}{x+3}$

$$5x^2 - 17x + 51 + \frac{-152}{x+3}$$

$$\begin{array}{r|l} x+3 & 5x^3 - 2x^2 + 0x + 1 \\ & -(5x^3 + 15x^2) \\ \hline & -17x^2 + 0x \\ & -(-17x^2 - 51x) \\ \hline & 51x + 1 \\ & -(51x + 153) \\ \hline & -152 \end{array}$$

2. Dividing By A Polynomial - Synthetic Division

ex: Divide using synthetic division.

a) $\dfrac{x^2+2x+7}{x-8}$

$$\begin{array}{r|rrr} 8 & 1 & 2 & 7 \\ & \downarrow & 8 & 80 \\ \hline & 1 & 10 & 87 \end{array}$$

$$1x+10+\frac{87}{x-8}$$

1) set den. = 0

2) Put this value on the 'side' of a box

3) put the coefficients of the numerator.

ex: Divide using synthetic division.

b) $\dfrac{x^2+3x-40}{x+5}$

$$\begin{array}{r|rrr} -5 & 1 & 3 & -40 \\ & & -5 & 10 \\ \hline & 1 & -2 & -30 \end{array}$$

$$\boxed{1x-2+\frac{-30}{x+5}}$$

ex: Divide using synthetic division.

c) $\dfrac{x^2-4}{x-1}$

$$x+1+\frac{-3}{x-1}$$

$$\begin{array}{c|ccc} 1 & 1 & 0 & -4 \\ & & 1 & 1 \\ \hline & 1 & 1 & -3 \end{array}$$

ex: Divide using any technique.

a) $\dfrac{x^3+2x^2+2x+9}{x^2+5}$

ex: Divide using any technique.

b) $\dfrac{x^3 + 7x^2 - x}{x + 3}$

Evaluating Polynomials

There are two ways to evaluate polynomial functions:

1. direct substitution

2. synthetic substitution

<u>Direct Substitution</u> (i.e. "PLUG IN")

ex: Find the indicated polynomial value using direct substitution.

a) $f(x) = x^2 - 5x + 2, \quad f(13) = ?$

ex: Find the indicated polynomial value using direct substitution.

b) $g(x) = x^3 + 4x^2 - 1, \quad g(6) = ?$

Synthetic Substitution - substitution using a chart of coefficients

*Before using synthetic substitution,
- the polynomial must be in standard form
- consider if all terms are present

ex: Find the indicated value using synthetic substitution.

a) $f(x) = x^2 - 5x + 2, \quad f(13) = ?$

ex: Find the indicated value using synthetic substitution.

b) $g(x) = x^3 + 4x^2 - 1, \quad g(6) = ?$

ex: Find the indicated value using synthetic substitution.

c) $m(x) = 5x^4 + 2x - 8, \quad m(-2) = ?$

REVIEW

ex: Perform the indicated operation.

a) $\left(x-5\right)-\left(x^2+2x+3\right)$

REVIEW

ex: Perform the indicated operation.

b) $(x-5)(x^2+2x+3)$

REVIEW

ex: Perform the indicated operation.

c) $\left(x^2+2x+3\right)^2$

REVIEW

ex: $6x^2 - 4x^5 + 1$

a) Write in standard form.

b) Identify the degree, leading coefficient and number of terms.

c)Classify the polynomial by degree and number of terms.

REVIEW

ex: Simplify.

$$\frac{15x^{-7}y^{9}z^{0}}{(3x)^{2}\left(xy^{-1}z^{5}\right)^{-2}}$$

REVIEW

ex: Factor completely.

a) $x^3 - 27$

b) $x^4 - 2x^2 - 8$