

$$11.) \log 1000 = \log_{10} 1000 = 3$$

$$20.) \log (.01) = \log_{10} \frac{1}{100} = -2$$

$$21.) \log_5 (-5) \text{ undefined}$$

$$\begin{aligned} 10^{\square} &= \frac{1}{100} \\ 10^{\square} &= 10^{-2} \end{aligned}$$

$$5^{\square} = -5$$

$$22) |\ln 1| = \log_e 1 = 0$$

$$e^{\square} = 1$$

$$9.) \log_{16} 2$$

$$\begin{aligned} 16^{\square} &= 2 \uparrow \\ 16^{1/4} &= \sqrt[4]{16} \end{aligned}$$

$$\log_{16} 4 = \frac{1}{2}$$

$$16.) \log_{1/2} 8 = -3$$

$$\frac{1}{2}^{\square} = 8 \uparrow$$
$$\frac{1}{2}^{\square} = 2^3$$
$$\frac{1}{2} = 2^{-1}$$
$$\cdot \left(\frac{1}{2}\right)^{-3} = 2^3$$

$$25.) \underline{\log_3 81} < \log_3 100 < \underline{\log_3 243}$$

4 and 5

$$3^1 = 3$$

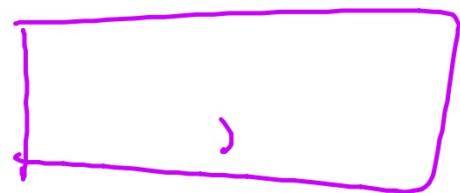
$$3^2 = 9$$

$$3^3 = 27$$

$$3^4 = 81$$

$$3^5 = 243$$

$$\underline{2} < \log_7 80 < \underline{3}$$



$$\begin{aligned}7^1 &= 7 \\7^2 &= 49 \\7^3 & \\7^4 &\end{aligned}$$

$$26.) \underline{1} < \log_5 10 < \underline{2}$$

$$\begin{array}{c} 5^0 = 1 \\ 5^1 = 5 \\ 5^2 = 25 \end{array}$$

27.)

$$\log_2 \frac{1}{10}$$

-3, -4

$$\begin{cases} 2^{-4} = \frac{1}{16} \\ 2^{-3} = \frac{1}{8} \end{cases}$$

$$2^{-2} = \frac{1}{4}$$

$$2^{-1} = \frac{1}{2}$$

$$2^0 = 1$$

$$2^1 = 2$$

$$2^2 = 4$$

## *Properties of Logarithms*



*Logarithms and exponentials are INVERSES!*

$$f(x) = \log_b x$$

$$g(x) = b^x$$

Evaluate.  $f(x) = \log_b x$        $g(x) = (b^x)$

a)  $(f \circ g)(x) = \log_b b^x = x$        $\log_2 16$   
                         $2^4$ .

b)  $(g \circ f)(x) = b^{\log_b x} = x$

## Inverse properties

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

$$\ln e^x = x$$

$$e^{\ln x} = x$$

$$\log_e e^x = x$$

*Evaluate.*

a)  $\underline{7^{\log_7 x}}$   
 $x$

b)  $\log_{\cancel{62}} 62^x$   
 $x$

c)  $\log_{\cancel{10}} 10^x$   
 $x$

d)  $e^{\ln 7}$   
 $e^{\log_e 7}$   
 $7$

e)  $\log_5 5^4$   
 $4$

f)  $8^{\log_8 6}$   
 $6$

$$b^m \cdot b^n = b^{m+n} \text{ (add)}$$

$$\frac{b^m}{b^n} = b^{m-n} \text{ (subtract)}$$

$$(b^m)^n = b^{mn} \text{ (multiply)}$$

## *Logarithm Properties*

Let  $b$ ,  $m$ , and  $n$  be positive numbers such that  $b \neq 1$

Product Property:  $\log_b(mn) = \log_b m + \log_b n$

Quotient Property:  $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property:  $\log_b m^n = n \cdot \log_b m$

*Logarithm properties are used to EXPAND and CONDENSE logarithmic expressions.*

*Expand. Simplify if possible.*

$$a) \log_3(a^2b^5) = \log_3 a^2 + \log_3 b^5 \\ = 2\log_3 a + 5\log_3 b$$

$$b) \log_5\left(\frac{a^2b^3}{c^4}\right) = \log_5 a^2 + \log_5 b^3 - \log_5 c^4 \\ = 2\log_5 a + 3\log_5 b - 4\log_5 c$$

$$c) \ln(e^4x^4) = \ln e^4 + \ln x^4 = 4\ln e + 4\ln x \\ 4(\ln e) + 4\ln x \\ 4 + 4\ln x$$

Expand. Simplify if possible.

$$\begin{aligned} d) \quad \log_2(4xy)^2 &= 2\log_2(4xy) \\ &= 2(\log_2 4) + 2\log_2 x + 2\log_2 y \\ &= 4 + 2\log_2 x + 2\log_2 y \end{aligned}$$

$$\begin{cases} \ln e = 1 \\ \ln 1 = 0 \end{cases}$$

$$\begin{aligned} e) \quad \ln\left(\frac{1}{ab^2c^3}\right) &= \ln 1 - (\ln a + \ln b^2 + \ln c^3) \\ &= \underset{0}{\ln 1} - \ln a - 2\ln b - 3\ln c \end{aligned}$$

$$\begin{aligned} f) \quad \log\sqrt{x^5y^6} &= \log(x^5y^6)^{1/2} = \frac{1}{2}\log x^5y^6 \\ &= \frac{1}{2}\log x^5 + \frac{1}{2}\log y^6 = \frac{5}{2}\log x + 3\log y \end{aligned}$$

*Expand. Simplify if possible.*

$$g) \ln\left(\frac{x+2}{x}\right) = \ln(x+2) - \ln x$$

$$h) \ln\left(\frac{x-2}{x+2}\right) = \ln x - \ln(x+2)$$

$$i) \log_2 \sqrt[3]{\frac{16a^5}{a+b}} = \log_2 \left( \frac{16a^5}{a+b} \right)^{\frac{1}{3}} = \frac{1}{3} \log_2 \left( \frac{16a^5}{a+b} \right)$$
$$= \frac{1}{3} \left( \log_2 16 + \log_2 a^5 - \log_2 (a+b) \right)$$
$$= \frac{4}{3} \log_2 16 + \frac{5}{3} \log_2 a - \frac{1}{3} \log_2 (a+b)$$

*Expand. Simplify if possible.*

$$\begin{aligned} j) \quad \log_3\left(\frac{81}{xy^2}\right) &= \log_3 81 - \log_3 x - \log_3 y^2 \\ &= 4 - \log_3 x - 2\log_3 y \end{aligned}$$

$$\begin{aligned} k) \quad \log_5\left(\frac{x^2 - 16}{125}\right) &= \log_5\left(\frac{(x+4)(x-4)}{125}\right) \\ &= \log_5(x+4) + \log_5(x-4) - 3 \end{aligned}$$

$$l) \quad \log_4\left(\frac{16x}{x-4}\right) = \cancel{\log_4}(e + \log_4 x - \log_4(x-4))$$

Condense.

$$a) 2\log_5 a + 3\log_5 b + 4\log_5 c$$

$$\log_5 a^2 + \log_5 b^3 + \log_5 c^4 = \log_5 (a^2 b^3 c^4)$$

$$b) 3\ln x + 2\ln y - 10\ln z$$

$$\ln x^3 + \ln y^2 - \ln z^{10} = \ln \left( \frac{x^3 y^2}{z^{10}} \right)$$

$$c) \log_3(x-3) - \log_3(x+2) = \log_3 \frac{x-3}{x+2}$$

*Condense.*

$$d) \log x + \log(x+1) = \log(x(x+1)) = \log(x^2+x)$$

$$e) \log x - 2\log(x+5) - \frac{1}{2}\log(x-1) = \log \frac{x}{(x+5)^2\sqrt{x-1}}$$

$$f) \frac{1}{2}\log x - \frac{1}{2}\log y - \frac{3}{2}\log z = \log \left( \frac{\sqrt{x}}{\sqrt{y}z^{3/2}} \right)$$