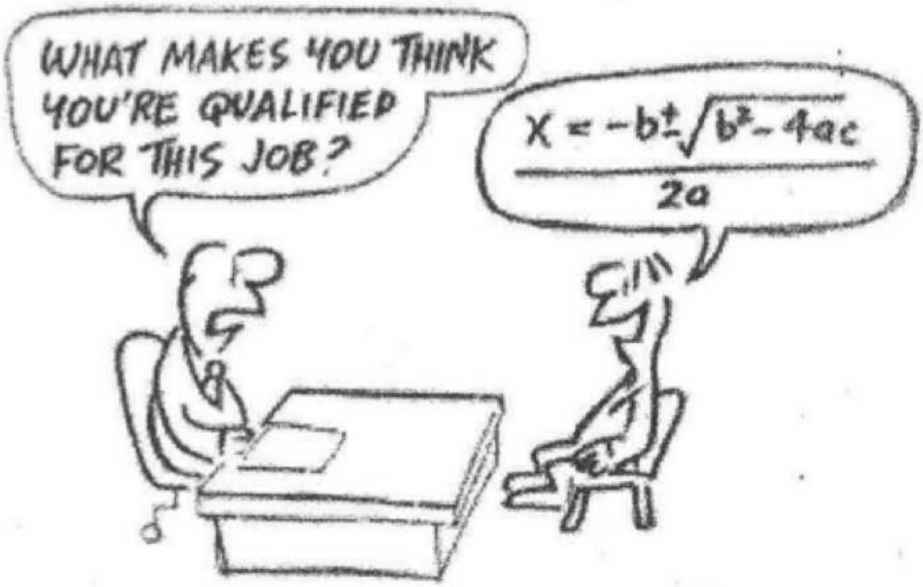
Graphing Quadratic Functions in Standard Form



Standard Form:

$$f(x) = ax^2 + bx + c$$

quadratic coefficient linear coefficient constant

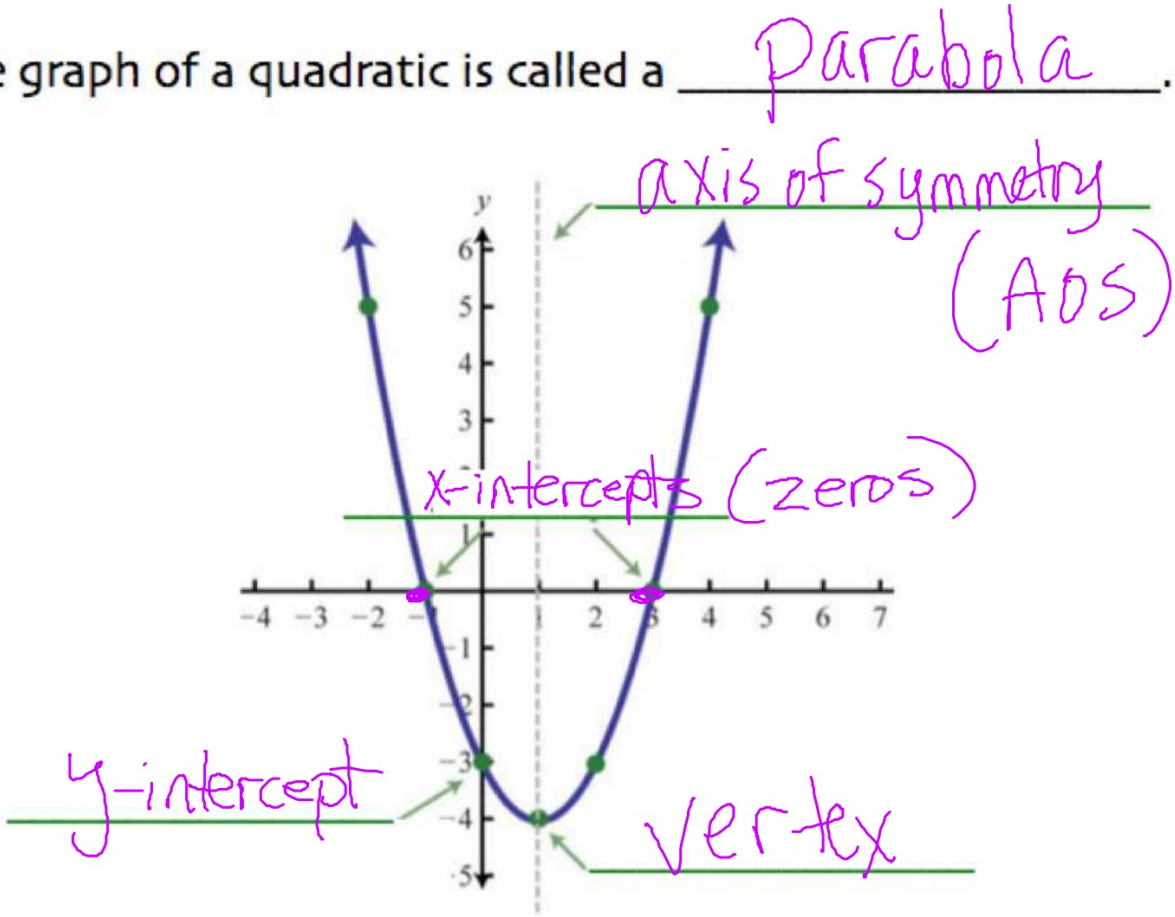
where:

a $\neq 0$

b $\in \mathbb{R}$

c $\in \mathbb{R}$

* The graph of a quadratic is called a parabola.



To graph a quadratic function in standard form:

- Find the vertex.

$$\boxed{\text{Vertex: } x = \frac{-b}{2a}}$$

- Plot at least two other points, one on each side of the vertex.

ex: Sketch, then state the vertex, axis of symmetry, y-intercept and domain and range in the indicated notation.

$a=3$ $b=-12$

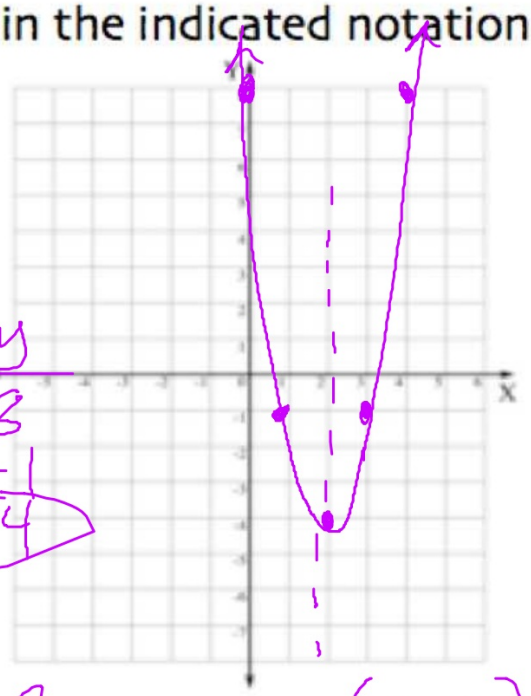
a) $y = 3x^2 - 12x + 8$

$x = \frac{-b}{2a} = \frac{-(-12)}{2(3)} = 2$

$y = 3(2)^2 - 12(2) + 8$

$y = 12 - 24 + 8 = -4$

x	y
0	8
1	-1
2	-4
3	-1
4	8



Vertex: $(2,-4)$ AOS: $x=2$ y-int: $(0,8)$

(INT) Domain: $(-\infty,\infty)$ Range: $[-4,\infty)$

$a = -1$ $b = -6$

$x = -2$; $-(-2)^2 - 6(-2) - 4$

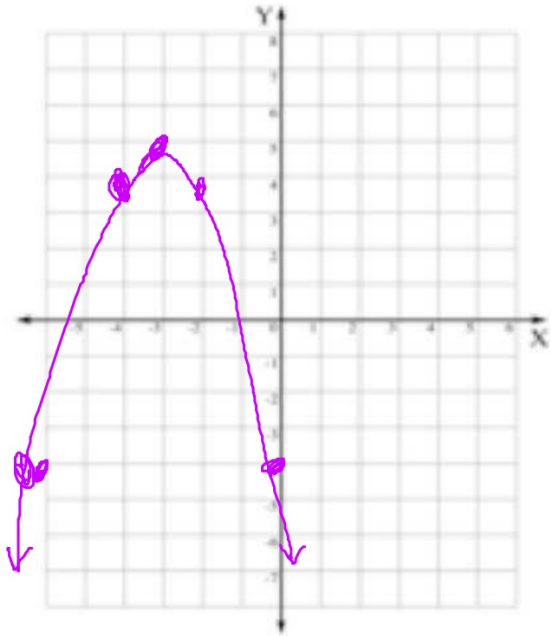
b) $y = -x^2 - 6x - 4$

$x = \frac{-b}{2a} = \frac{-(-6)}{2(-1)} = -3$

$y = -(-3)^2 - 6(-3) - 4$

$y = -9 + 18 - 4 = 5$

x	y
-6	-4
-4	4
* -3	5
-2	4
0	-4

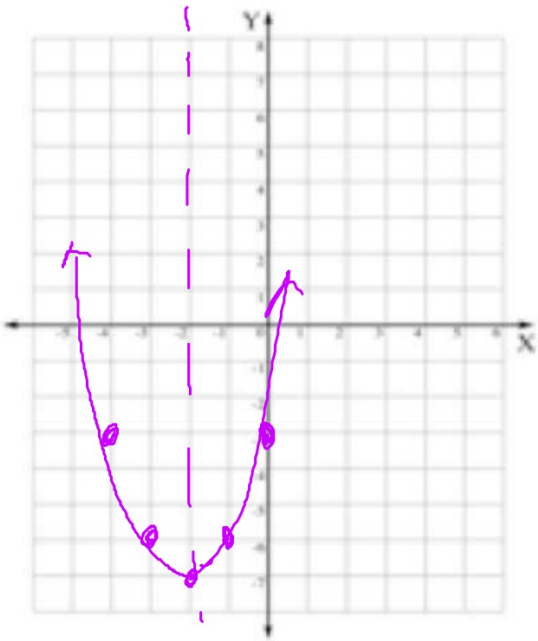


Vertex: $(-3, 5)$ AOS: $x = -3$ y-int: $(0, -4)$

(SET) Domain: $\{x \mid x \in \mathbb{R}\}$ $(-\infty, \infty)$ Range: $\{y \mid y \leq 5\}$ $(-\infty, 5]$

c) $y = x^2 + 4x - 3$

x	y
-1	-6
-2	-7
-3	-6



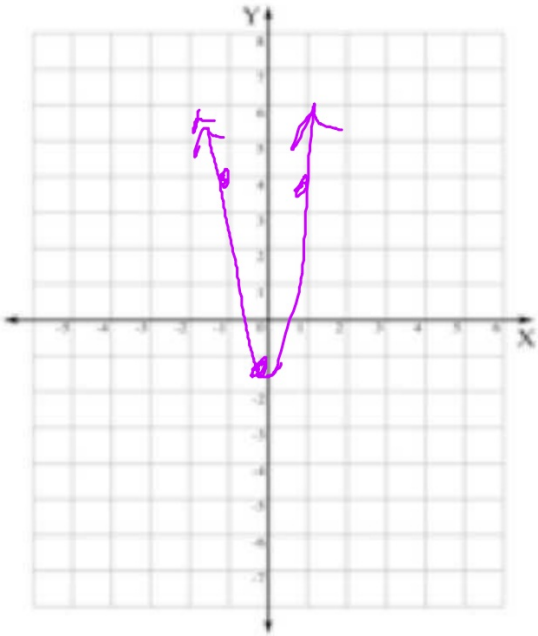
Vertex: (-2,-7) AOS: x=-2 y-int: (0,-3)

(INT) Domain: (-∞,∞) Range: [-7,∞)

d) $y = 5x^2 - 1$

$x = \frac{-0}{2(5)} = 0$

x	y
-1	4
0	-1
1	4



Vertex: (0, -1) AOS: x = 0 y-int: (0, -1)

~~(SET)~~ Int Domain: $(-\infty, \infty)$ Range: $[-1, \infty)$

e) $y = x^2 - x + 1$

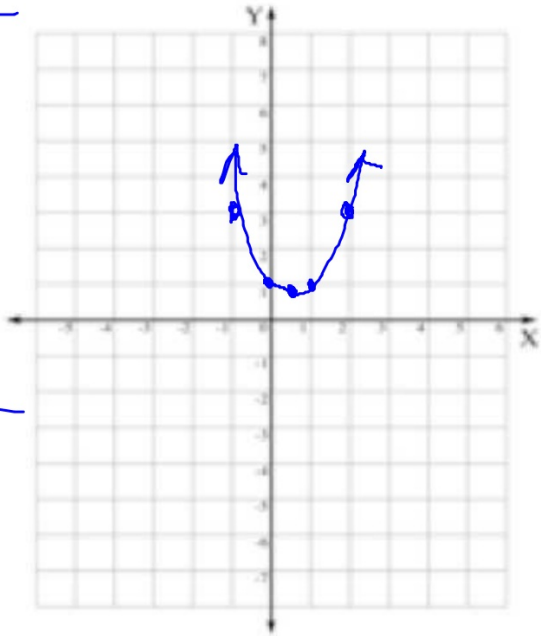
$$x = \frac{-(-1)}{2(1)} = \frac{1}{2}$$

$$y = \left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1$$

$$y = \frac{1}{4} - \frac{1 \times 2}{2 \times 2} + \frac{4}{4} = \frac{1 - 2 + 4}{4}$$

$$= \frac{3}{4}$$

X	y
-1	3
0	1
1/2	3/4
1	1
2	3



Vertex: $\left(\frac{1}{2}, \frac{3}{4}\right)$ AOS: $x = \frac{1}{2}$ y-int: $(0, 1)$

~~(SET)~~ Int Domain: $(-\infty, \infty)$ Range: $[3/4, \infty)$

ex: Consider the graph of: $y = ax^2 + bx + c$

a) When does the graph open up?

$a > 0$

b)When does the graph open down?

$a < 0$

c) What is the axis of symmetry?

$x = \frac{-b}{2a}$

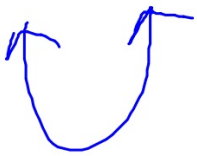
Maximum and Minimum Values

* The maximum or minimum of a parabola always occurs at the vertex.

*The maximum or minimum value is the y-coordinate of the vertex.

ex: State the maximum or minimum value.

a) $y = 3x^2 - 12x + 8$



$$x = \frac{-(-12)}{2(3)} = 2$$

$$y = 3 \cdot 2^2 - 12 \cdot 2 + 8 = -4 \leftarrow \text{min value}$$

b) $y = -x^2 - 6x - 4$

$$x = \frac{-(-6)}{2(-1)} = -3$$

$$y = -(-3)^2 - 6(-3) - 4$$

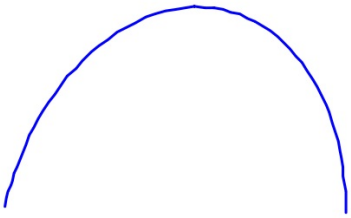
$$= -9 + 18 - 4 = 5$$

opens down

$\boxed{\text{max value: } 5}$

ex: State the maximum or minimum value.

c) $y = -3x^2 + 12x - 7$



$$x = \frac{-12}{2(-3)} = 2$$

$$y = -3 \cdot 2^2 + 12 \cdot 2 - 7$$

$$= -12 + 24 - 7$$

$$= 5 \text{ Max}$$

ex: Without graphing, consider the function: $y = -\frac{1}{2}x^2 + 3$

a) What is the direction of opening?

down

b)What is the axis of symmetry?

$x = 0$

c) What is the maximum/minimum value?

3

d) State the domain and range in interval notation.

$(-\infty, \infty)$ $(-\infty, 3]$

ex: Write the quadratic function in standard form.

a) $y = 5(x-1)^2 + 4$

$y = 5(x-1)(x-1) + 4$

$y = 5(x^2 - 2x + 1) + 4$

$y = 5x^2 - 10x + 5 + 4$

$y = 5x^2 - 10x + 9$

ex: Write the quadratic function in standard form.

b) $y = 3(x+5)(x-2)$

$y = 3(x^2 + 3x - 10)$

$y = 3x^2 + 9x - 30$

ex: Write the quadratic function in standard form.

c) $y = 2(x+7)^2 - 1$

ex: Write the quadratic function in standard form.

d) $y = -4(x-5)(x+5)$