

$$14) (2x+1)^2(2x^2+5x-1)=0$$

$$2x+1=0$$

$$x = \frac{-1}{2}$$

Mult. of 2

$$2x^2+5x-1=0$$

$$x = \frac{-5 \pm \sqrt{25-4(2)(-1)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{33}}{4}$$

$$\begin{aligned} \text{*15)} \quad & x^6 - 64 = 0 \\ & (x^3 - 8)(x^3 + 8) = 0 \\ & (x-2)(x^2 + 2x + 4)(x+2)(x^2 - 2x + 4) = 0 \\ & \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \\ & x=2 \qquad \qquad \qquad x=-2 \\ & \qquad \qquad \qquad / \end{aligned}$$

$$\begin{aligned}
 & |2.) \quad 27x^3 + 54x^2 = x + 2 \\
 & \underbrace{27x^3 + 54x^2 - x - 2}_{} = 0 \\
 & 27x^2(x+2) - 1(x+2) = 0 \\
 & (27x^2 - 1)(x+2) = 0 \\
 & 27x^2 = 1 \\
 & \sqrt{x^2} = \sqrt{\frac{1}{27}} \\
 & x = \pm \frac{1}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\
 & \qquad \qquad \qquad -2 \\
 & \qquad \qquad \qquad \Rightarrow \pm \frac{\sqrt{3}}{9}
 \end{aligned}$$

$$\text{II.) } x^7 - x^4 = 0$$

$$x^4(x^3 - 1) = 0$$

$$x^4(x-1)(x^2+x+1) = 0$$

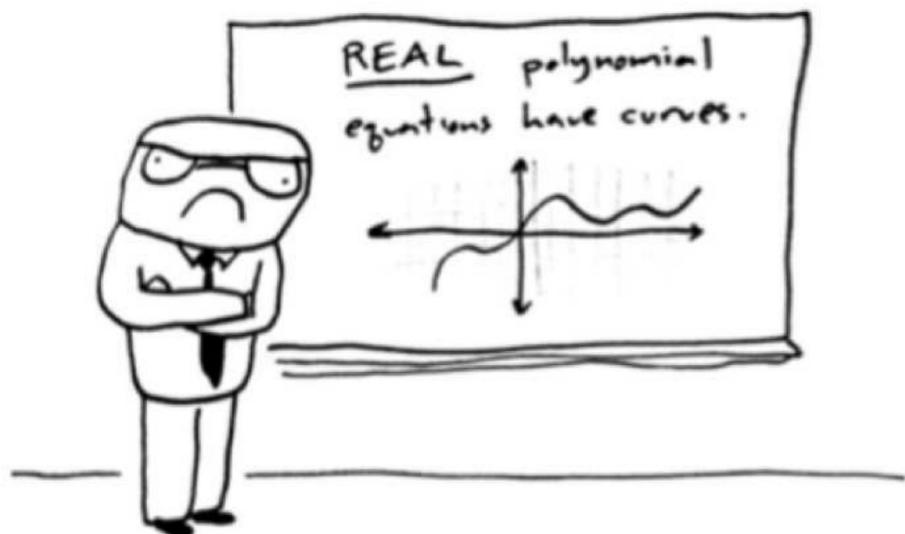
$$x^4 = 0 \quad | \quad x-1 = 0 \quad | \quad x^2+x+1 = 0$$

$$x = 0$$

mult. of 4

$$18.) \sqrt{2}, -\sqrt{2}, 4$$
$$\begin{array}{c} \downarrow \quad \downarrow \\ (x-\sqrt{2})(x+\sqrt{2})(x-4)=0 \end{array}$$
$$(x^2-2)(x-4)=0$$
$$x^3-4x^2-2x+8=0$$

Apply the Remainder and Factor Theorems
Find Rational Zeros
Introduction to the Rational Zero Theorem



Toothpaste For Dinner.com

The Factor Theorem

A polynomial $g(x)$ is a factor of $f(x)$ if...

1. $\frac{f(x)}{g(x)}$ has a remainder of 0.

2. k is a zero of $g(x)$ and $f(k) = 0$.

$$\frac{12}{2} = 6 \quad \frac{12}{5}$$

ex: Is $g(x)$ a factor of $f(x)$? Yes! because the remainder is 0.

a) $g(x) = x - 5$, $f(x) = x^3 - 7x^2 + 7x + 15$

$$\begin{array}{r|rrrr} 5 & 1 & -7 & 7 & 15 \\ \downarrow & 5 & -10 & -15 \\ \hline 1 & -2 & -3 & 0 \end{array}$$

ex: Is $g(x)$ a factor of $f(x)$? **No!** The remainder is not 0.

b) $g(x) = x + 7$, $f(x) = x^2 - 9$

$$\begin{array}{r} \boxed{1 \quad 0 \quad -9} \\ \underline{-7} \quad 49 \\ 1 \quad -7 \quad \underline{40} \end{array}$$

ex: Factor $f(x)$ completely given one of its factors.

a) $f(x) = 15x^3 + x^2 - 22x - 8$; $x+1$

$$\begin{array}{r} -1 \left| \begin{array}{cccc} 15 & 1 & -22 & -8 \\ -15 & 14 & 8 \\ \hline 15 & -14 & -8 & 0 \end{array} \right. \end{array}$$

$$(x+1)(3x-4)(5x+2) = 0$$

$$15x^2 - 14x - 8 = 0$$

$$(3x-4)(5x+2) = 0$$

ex: Factor $f(x)$ completely given one of its factors.

b) $f(x) = x^3 - 7x^2 + 7x + 15$; $x - 3$

$$\begin{array}{r} 3 | 1 \ -7 \ 7 \ 15 \\ \quad \quad 3 \ -12 \ -15 \\ \hline 1 \ -4 \ -5 \ 0 \end{array}$$

$(x-3)(x-5)(x+1)=0$

$|x^2 - 4x - 5$
 $(x-5)(x+1)$

ex: Find the zeros of $f(x)$ given one of its zeros.

$$f(x) = x^3 + 6x^2 + 9x + 4; \quad -4$$

$$0 = x^3 + 6x^2 + 9x + 4$$

$$\begin{array}{r} 1 \ 6 \ 9 \ 4 \\ -4 \ -8 \ -4 \\ \hline 1 \ 2 \ 1 \ 0 \end{array}$$

$$\begin{aligned} &x^2 + 2x + 1 \\ &(x+1)(x+1) = 0 \end{aligned}$$

$$\begin{array}{c} x = -1 \\ \text{mult. } 2 \end{array}$$

The Rational Zero Theorem

If $f(x)$ is a polynomial then every rational zero of $f(x)$ comes in the form of...

$$\frac{p}{q} : \frac{\text{factor of constant term}}{\text{factor of leading coefficient}}$$

$$5x^3 - 2x^2 - 7x + 4$$

\uparrow \uparrow
leading constant

$$\left. \begin{array}{l} p = 4 \\ q = 5 \end{array} \right\}$$

ex: List the possible rational zeros.

a) $f(x) = 2x^3 - 7x^2 + 9$

P : $\pm 1, \pm 3, \pm 9$

q : $\pm 1, \pm 2$

$\frac{P}{q} : \pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 9, \pm \frac{9}{2}$

ex: List the possible rational zeros.

b) $f(x) = 4x^4 - x^3 - 7x^2 + 4x - 2$

$$P: \pm 1, \pm 2$$

$$q: \pm 1, \pm 2, \pm 4$$

$$\frac{P}{q}: \pm 1, \pm \frac{1}{2}, \pm 2, \pm \frac{1}{4}$$