

## Analyzing Polynomial Functions

### Factoring Review



HW:

*Analyzing polynomial functions WKST*

## REVIEW

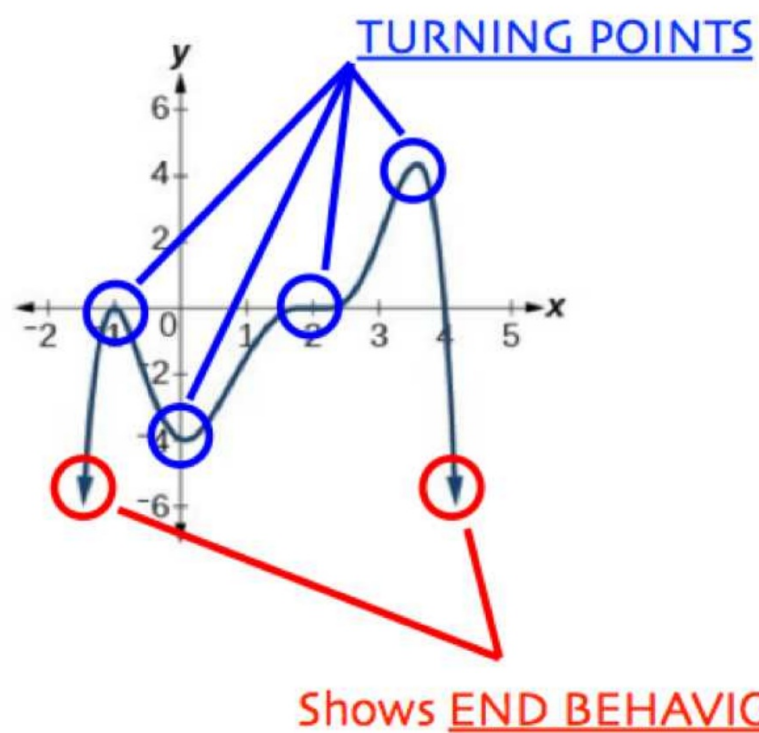
What is a polynomial function?

A polynomial function is an expression involving one or more monomials. Polynomial functions have variables with whole exponents, real coefficients and contain no division by variables. The degree of a polynomial is the largest exponent (attached to a variable). The leading coefficient is the coefficient of the term that defines the degree.

$$f(x) = 2x^2 - 4x + \frac{1}{7} \qquad f(x) = 0$$

$$f(x) = \frac{x^3}{5} \qquad f(x) = 3x^5 - x^4 + 5x - 1$$

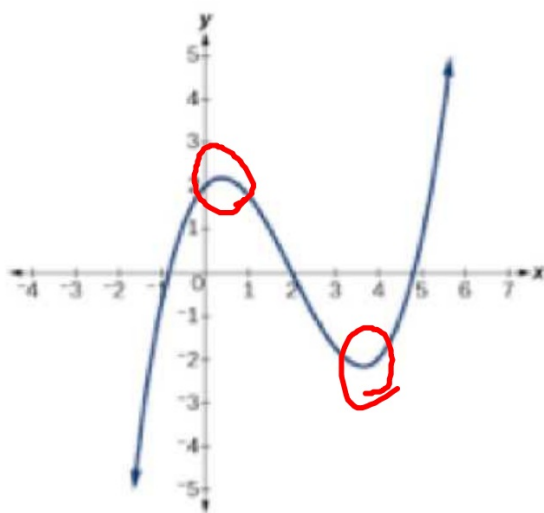
## Graphs of Polynomial Functions



\*A turning point can occur at a maximum, minimum or at a "flat point."

ex: Using the graph determine the number of turning points.

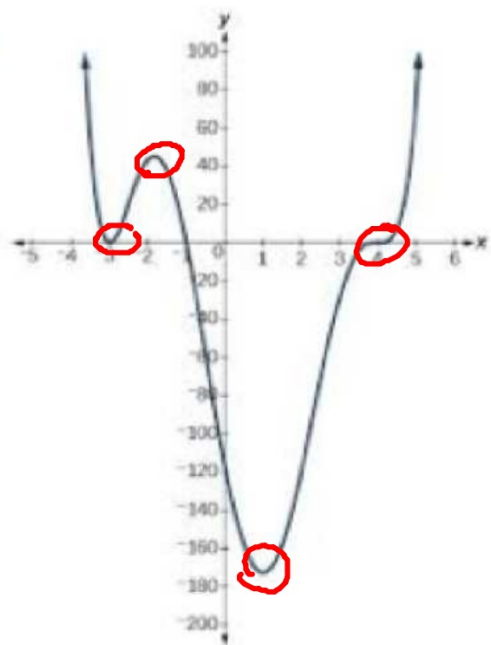
a)



2

ex: Using the graph determine the number of turning points.

b)



4

## Polynomial Degrees and Number of Turning Points

Polynomial Type	Degree	Maximum Number of Turning Points
Constant <i>number</i>	<i>0</i>	<i>0</i>
Linear	<i>1</i>	<i>0</i>
Quadratic	<i>2</i>	<i>1</i>
Cubic	<i>3</i>	<i>2</i>
$n^{\text{th}}$ Degree Polynomial	<i>n</i>	<i>n-1</i>

ex: Determine the degree and state the maximum number of turning points.

a)  $f(x) = 2x^3 + 5x^2 - 9$

3 ; 2

b)  $f(x) = 9x^4 - 6x^5$

5 ; 4



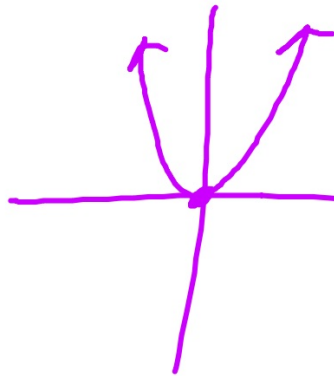
ex: Determine the degree and state the maximum number of turning points.

c)  $f(x) = (3x-1)^2 = (3x-1)(3x-1)$   
 $9x^2 - 6x + 1$   
2; 1

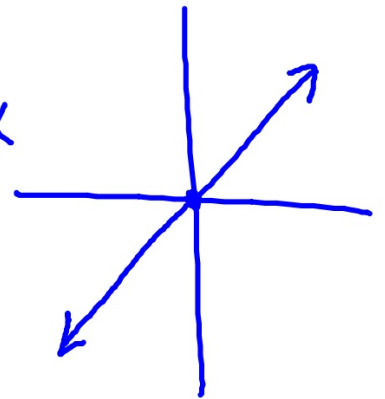
d)  $f(x) = (x^2 - 5)(2x + 7)^3$   
2      3  
5; 4

$X^2 \cdot X^3$   
 $X^5$

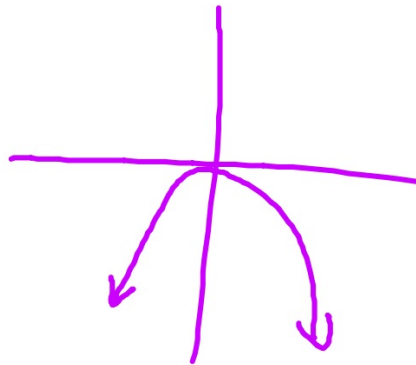
$$y = x^2$$



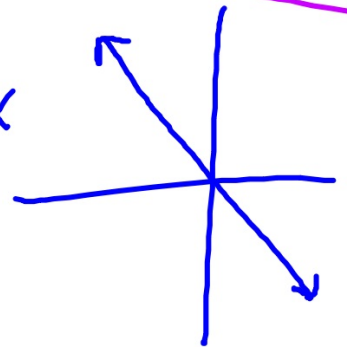
$$y = x$$



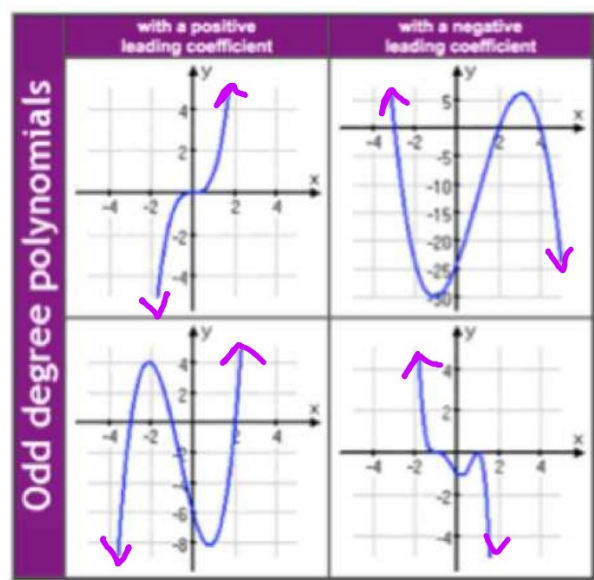
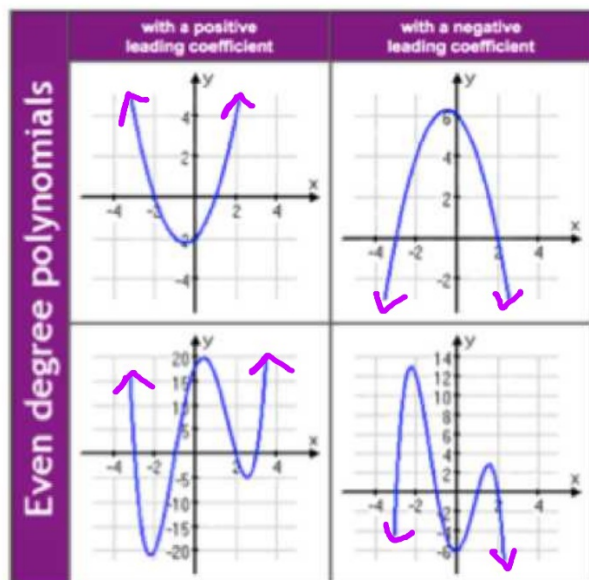
$$y = -x^2$$



$$y = -x$$



## End Behavior, Degrees & Leading Coefficients



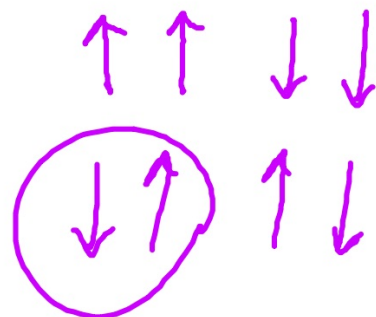
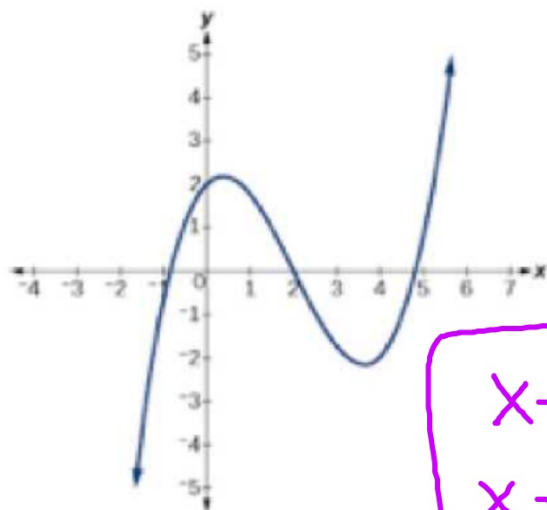
## Stating End Behavior

$$x \rightarrow -\infty, \quad y \rightarrow \underline{\hspace{2cm}}$$

$$x \rightarrow \infty, \quad y \rightarrow \underline{\hspace{2cm}}$$

ex: Determine the end behavior of each polynomial.

a)



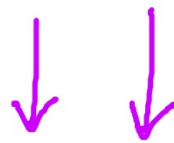
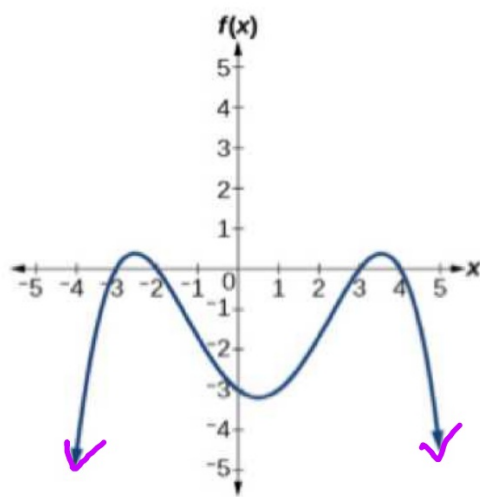
$x \rightarrow -\infty$   $y \rightarrow -\infty$  (down)

$x \rightarrow \infty$   $y \rightarrow \infty$  (up)

State  
end behavior

ex: Determine the end behavior of each polynomial.

b)



$$x \rightarrow -\infty$$

$$y \rightarrow -\infty$$

$$x \rightarrow \infty$$

$$y \rightarrow -\infty$$

ex: Determine the end behavior of each polynomial.

c)  $f(x) = \underline{\underline{2x^3}} + 5x^2 - 9$

odd degree  
pos. l.c.

↓ ↑

$$x \rightarrow -\infty \quad y \rightarrow -\infty$$

$$x \rightarrow \infty \quad y \rightarrow \infty$$

ex: Determine the end behavior of each polynomial.

d)  $f(x) = 9x^4 - \underline{\underline{\underline{6x^5}}}$

odd degree

neg. l.c.

↑ ↓

$x \rightarrow -\infty$	$y \rightarrow \infty$	↑
$x \rightarrow \infty$	$y \rightarrow -\infty$	↓



ex: Determine the end behavior of each polynomial.

e)  $f(x) = (3x - 1)^2$   
 $9x^2$

even degree

pos. l.c.

↑ ↑

$$x \rightarrow -\infty \quad y \rightarrow \infty$$

$$x \rightarrow \infty \quad y \rightarrow \infty$$

ex: Determine the end behavior of each polynomial.

f)  $f(x) = (x^2 - 5)(2x + 7)^3$

$$x^2 \cdot x^3 = x^5$$

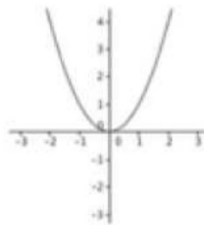
odd degree  
pos. l.c.



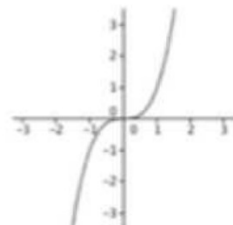
$$\begin{array}{ll} x \rightarrow -\infty & y \rightarrow -\infty \\ x \rightarrow \infty & y \rightarrow \infty \end{array}$$

## Bouncing and Crossing Zeros

In the graph below the graph "bounces" off the x-axis at  $x=0$ .



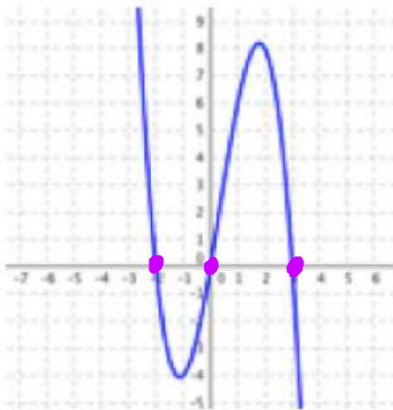
In the graph below the graph "crosses" the x-axis at  $x=0$ .



ex: Using the graph or the equation of the polynomial function,

1. Find the zeros. State the multiplicity if greater than 1.
2. Determine whether the graph "crosses" the x-axis or "bounces" off the x-axis at each zero.

a)  $f(x) = -x(x+2)(x-3)$

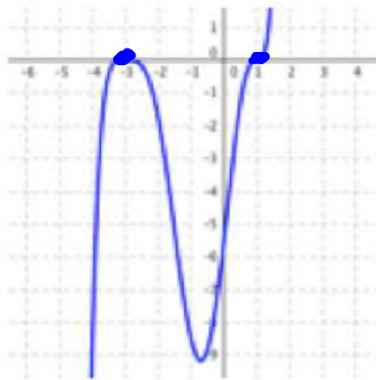


$x = 0$   
 $x = -2$   
 $x = 3$   
All mult. of 1  
All crossing.

ex: Using the graph or the equation of the polynomial function,

1. Find the zeros. State the multiplicity if greater than 1.
2. Determine whether the graph "crosses" the x-axis or "bounces" off the x-axis at each zero.

b)  $f(x) = \frac{1}{15}(x+3)^4(x-1)^3$

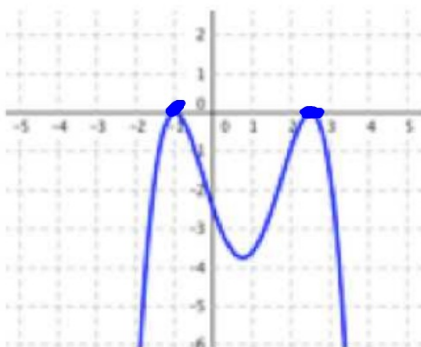


$x = -3$ , mult. of 4 bounce  
 $x = 1$ , mult. of 1 cross

ex: Using the graph or the equation of the polynomial function,

1. Find the zeros. State the multiplicity if greater than 1.
2. Determine whether the graph "crosses" the x-axis or "bounces" off the x-axis at each zero.

c)  $f(x) = -\frac{1}{10}(2x-5)^2(x+1)^2$



$x = -1$  mult. of 2

$x = 2.5$  mult. of 2

both bounce

- A graph "crosses" the x-axis at a zero if the multiplicity of that zero is odd.
- A graph "bounces" off the x-axis at a zero if the multiplicity of that zero is even.

## REVIEW

ex: Factor completely, if possible.

a)  $4x^3 - 25x^2 + 25x$

$$x(4x^2 - 25x + 25)$$

$$x(4x - 5)(x - 5)$$



## REVIEW

ex: Factor completely, if possible.

b)  $x^2 + 9$

*does not factor*

## REVIEW

ex: Factor completely, if possible.

$$c) 2x^3 - 3x^2 + 10x - 15$$

$$(x^2 + 5)(2x - 3)$$

## REVIEW

ex: Factor completely, if possible.

d)  $8x^3 - 1$

$$(2x - 1)(4x^2 + 2x + 1)$$

## REVIEW

ex: Factor completely, if possible.

e)  $2x^2 - 32$

$$2(x - 4)(x + 4)$$

## REVIEW

ex: Factor completely, if possible.

f)  $-5x^2 + 18x - 9$

$$-(5x - 3)(x - 3)$$

## REVIEW

ex: Factor completely, if possible.

g)  $2x^4 + 7x^2 + 6$

$$(2x^2 + 1)(x^2 + 3)$$

## REVIEW

ex: Factor completely, if possible.

h)  $x^5 - x^3 + 64x^2 - 64$

$$(x + 4)(x^2 - 4x + 16)(x - 1)(x + 1)$$

## REVIEW

ex: Factor completely, if possible.

i)  $x^4 + 4x^2 + 5$

*does not factor*