

Taylor and Maclaurin Series

I. Write the first four nonzero terms of the Taylor series for $f(x)$ about $x=c$.

1. $f(x) = e^x, c = 1$

2. $f(x) = \cos x, c = \frac{\pi}{4}$

3. $f(x) = \frac{1}{x}, c = 1$

II. Use the basic 5 to write the first four nonzero terms and the general term of the Maclaurin series for $f(x)$.

4. $f(x) = e^{x/2}$

5. $f(x) = \ln(1+x)$

6. $f(x) = \sin 3x$

7. $f(x) = \cos x^{3/2}$

8. $f(x) = x \sin x$

III. Use the basic 5 to write the first four nonzero terms and the general term for the Taylor series for f about $x=0$.

9. $f(x) = \frac{\cos(3x)}{x}$

10. $f(x) = x^2 e^{-x}$

11. $f(x) = \cos^2 x$ (first four terms only)

12. $f(x) = \sin(x^2)$

13. $f(x) = \int_0^x (e^{-t^2} - 1) dt$

14. $f(x) = \frac{\sin x}{x}$

15. $f(x) = \frac{e^{x/2} - 1}{x}$

IV. Use a power series to find the indicated limits.

16. $\lim_{x \rightarrow 0} \cos^2 x$ (Use your series from #11)

17. $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ (Use your series from #14)

V. Use a power series to approximate the integral value with an error less than 0.001.

18. $\int_0^1 \sin(x^2) dx$ (Use your series from #12)

19. $\int_0^1 x^2 e^{-x} dx$ (Use your series from #10)

VI. Find the indicated value if $f(x) = 1 - x + x^2 - x^3 + \dots + (-1)^n x^n + \dots$

20. $f'(0)$

21. $f^{(4)}(0)$

22. $f^{(9)}(0)$

VII. The function f has a Taylor series about $x=3$ that converges to $f(x)$ for all x in the interval of

convergence. The n th derivative of f at $x=3$ is given by $f^{(n)}(3) = \frac{(-1)^n n!}{5^n n^2}$ for $n \geq 1$, and $f(3) = 0$.

23. Find the first four nonzero terms and general term for the Taylor series about $x=3$.

24. Find the radius of convergence of the Taylor series for f about $x=3$. Show the work that leads to your answer.

ANSWERS

1. $f(x) = e^x = e + e(x-1) + \frac{e(x-1)^2}{2!} + \frac{e(x-1)^3}{3!} + \dots$

16. 1

2. $f(x) = \cos x = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}(x-\pi/4)}{2} + \frac{\sqrt{2}(x-\pi/4)^2}{2 \cdot 2!} + \frac{\sqrt{2}(x-\pi/4)^3}{2 \cdot 3!} + \dots$

17. 1

3. $f(x) = \frac{1}{x} = 1 - (x-1) + (x-1)^2 - (x-1)^3 + \dots$

18. 0.310

4. $f(x) = e^{x/2} = 1 + \frac{x}{2} + \frac{x^2}{2^2 \cdot 2!} + \frac{x^3}{2^3 \cdot 3!} + \dots + \frac{x^n}{2^n n!} + \dots$

19. 0.160

5. $f(x) = \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + \frac{(-1)^n x^{n+1}}{n+1} + \dots$

20. -1

6. $f(x) = \sin 3x = 3x - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} - \frac{(3x)^7}{7!} + \dots + \frac{(-1)^n (3x)^{2n+1}}{(2n+1)!} + \dots$

21. 4!

7. $f(x) = \cos x^{3/2} = 1 - \frac{x^3}{2!} + \frac{x^6}{4!} - \frac{x^9}{6!} + \dots + \frac{(-1)^n x^3}{(2n)!} + \dots$

22. -9!

8. $f(x) = x \sin x = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + \dots + \frac{(-1)^n x^{2n+2}}{(2n+1)!} + \dots$

$$f(x) = -\frac{x-3}{5} + \frac{(x-3)^2}{5^2 \cdot 2^2}$$

9. $f(x) = \frac{1}{x} - \frac{3^2 x}{2!} + \frac{3^4 x^3}{4!} - \frac{3^6 x^5}{6!} + \dots + \frac{(-1)^n 3^{2n} x^{2n-1}}{(2n)!} + \dots$

$$23. -\frac{(x-3)^3}{5^3 \cdot 3^2} + \frac{(x-3)^4}{5^4 \cdot 4^2} - \dots$$

10. $f(x) = x^2 - x^3 + \frac{x^4}{2!} - \frac{x^5}{3!} + \dots + \frac{(-1)^n x^{n+2}}{n!} + \dots$

$$+ \frac{(-1)^{n+1} (x-3)^{n+1}}{5^{n+1} \cdot (n+1)^2} + \dots$$

24. 5

11. $f(x) = 1 - \frac{(2x)^2}{2 \cdot 2!} + \frac{(2x)^4}{2 \cdot 4!} - \frac{(2x)^6}{2 \cdot 6!} + \dots$

12. $f(x) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots + \frac{(-1)^n x^{4n+2}}{(2n+1)!} + \dots$

13. $f(x) = -\frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} - \frac{x^7}{7 \cdot 3!} + \frac{x^9}{9 \cdot 4!} - \dots + \frac{(-1)^{n+1} x^{2n+3}}{(2n+3)(n+1)!} + \dots$

14. $f(x) = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \dots$

15. $f(x) = \frac{1}{x} + \frac{1}{2} + \frac{x}{4 \cdot 2!} + \frac{x^2}{8 \cdot 3!} + \dots + \frac{x^n}{2^{n+1} (n+1)!} + \dots$