6.4 Estimating a Population Mean μ : σ Not Known

(This section is much more realistic than 6.3)

Important Properties of the Student *t* Distribution

Slide 72

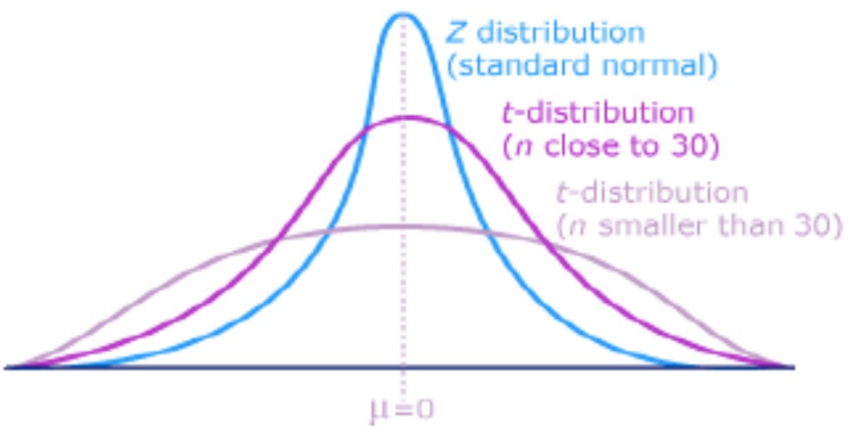
1. The Student *t* distribution is different for different sample sizes (see Figure 6-5 for the cases $n = 3$ and $n = 12$).

2. The Student *t* distribution has the same general symmetric bell shape as the normal distribution but it reflects the greater variability (with wider distributions) that is expected with small samples.

3. The Student *t* distribution has a mean of $t = 0$ (just as the standard normal distribution has a mean of $z = 0$).

4. The standard deviation of the Student *t* distribution varies with the sample size and is greater than 1 (unlike the standard normal distribution, which has a $\sigma = 1$).

5. As the sample size *n* gets larger, the Student *t* distribution gets closer to the normal distribution.


Z distribution
(standard normal)
t-distribution
(n close to 30)
t-distribution
(n smaller than 30)
μ=0

The methods covered in 6.4 are the ones used often.
They are realistic & practical.

USUALLY YOU DON'T KNOW σ

WHEN YOU ARE LOOKING FOR μ

The following assumptions need to be true in order to use the methods you're about to learn:

1) The sample is a simple random sample (SRS)
2) The sample is normally distributed and/or n> 30

The sample mean $\bar{x}$ is the best point estimate of the population mean μ.

As with any best point estimate, we don't know how good the estimate is so we're better off using a confidence interval.

***** When σ is not known, we use t instead of z to find the confidence interval.

To find the critical value, $t_{\alpha/2}$ you need to know a the number of degrees of freedom & to use table A3.

Since confidence intervals are two-sided, we will use "Area in two tails" on the chart.

$t_{\alpha/2}$

degrees of freedom = n - 1

The number of degrees of freedom for a sample of data is the # of sample values that can vary after certain restrictions have been placed on all the data values. (You'll understand better after I give you an example.)

$df = n - 1$

$n = 5$

$df = 4$

$\boxed{11}$ $\boxed{12}$ $\boxed{-17}$ $\boxed{25}$ $\boxed{-21}$

$= 10$

$n = 19$ 95% $df = 18$ $\alpha = .05$ $t_{\alpha/2} = 2.101$

$n = 15$ 90% $df = 14$ $\alpha = .10$ $t_{\alpha/2} = 1.761$

$\alpha = 1 - .90$

$n = 43$ 99% $df = 42$ $\alpha = .01$ ← pick the lower df $t_{\alpha/2} = 2.704$

$n = 55$ 80% $df = 54$ $\alpha = .20$ $t_{\alpha/2} = 1.299$

$df = 27$ 99% $df = 27$ $\alpha = .01$ $t_{\alpha/2} = 2.771$

Margin of Error E for Estimate of μ

Based on an Unknown σ and a Small Simple Random Sample from a Normally Distributed Population

Formula 6-6

$$\boxed{E = t_{\alpha/2} \frac{s}{\sqrt{n}}}$$

$\bar{x} - E < \mu < \bar{x} + E$

or

$(\bar{x} - E, \bar{x} + E)$

where $t_{\alpha/2}$ has n – 1 degrees of freedom.

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Confidence Interval (or Interval Estimate) for Population Mean μ when σ is known

$$\boxed{\bar{x} - E < \mu < \bar{x} + E}$$

$$\bar{x} \pm E$$

$$\boxed{(\bar{x} - E, \bar{x} + E)}$$

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Before you find the confidence interval for μ you need to see what information you have. No matter what you need a simple random sample that is either normally distributed and/or n>30.

If you know σ, use the margin of error formula involving Z.

If you don't know σ, use the margin of error formula involving t.

After you find E, the process for finding the confidence interval is the same.

σ

Sigma known: z or neither

Sigma not known: t or neither

σ

If possible, which critical value (z or t) would you use to find the confidence interval for .

1) $n = 28$, $\bar{x} = 400$, $s = 16$, the population is skewed t or neither

2) $n = 32$, $\bar{x} = 400$, $s = 16$, the population is skewed t or neither

3) $n = 32$, $\bar{x} = 400$, $\sigma = 18$, the population is skewed z or neither

4) $n = 8$, $\bar{x} = 400$, $\sigma = 18$, the population is normally distributed z or neither

z : normal

Find the confidence interval for μ using the given info. Assume that the population has a normal dist.

16) 95% confidence; n = 15; $\bar{x}$ = 496; s = 108

95%

$\alpha = .05$

$df = n-1$

$= 15-1$

$= 14$

$t_{\alpha/2} = 2.145$

$$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{15}}$$

$$= 2.145 \cdot \frac{108}{\sqrt{15}} = 59.8$$

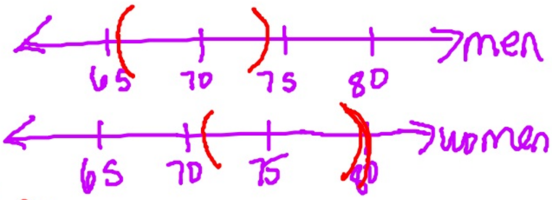
$$(436.2, 555.8)$$

I am 95% confident that the true mean of math SAT scores for women is between 436.2 and 555.8.

Pulse Rates for men and women(95% conf.)

Men	Women
$\bar{x} = 69.4$	$\bar{x} = 76.3$
$s = 11.3$	$s = 12.5$
$n = 40$	$n = 40$
$E = 2.024 \cdot \frac{11.3}{\sqrt{40}}$	$E = 2.024 \cdot \frac{12.5}{\sqrt{40}}$
$E = 3.6$	$E = 4.0$
$(65.8, 73)$	$(72.3, 80.3)$

Is there a difference between the pulse rates of men and women?



No, there is no significant difference.
The conf. int. overlap.

Construct a 95% confidence interval. Assume the population is normally distributed.

ACT scores for 20 randomly selected students.

26	22	23	12	19	25	23	21	25	10
20	22	23	21	14	20	24	23	26	17

$\bar{x} = 20.8$

$s = 4.5$

$t_{\alpha/2} = 2.093$

$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

$= 2.093 \cdot \frac{4.5}{\sqrt{20}}$

$E = 2.1$

$(18.7, 22.9)$

Construct a 99% confidence interval. Assume the population is normally distributed.

GPA for 14 randomly selected college students.

2.3	3.3	2.6	1.8	0.2	3.1	4.0
0.7	2.3	2.0	3.4	1.3	2.6	2.6