CHAPTER 6–CONFIDENCE INTERVALS

$$34\% \pm 4\%$$

30% – 38%

point estimate

$\hat{p}$

margin of error

E

6.2 Estimating a Population Proportion

In this section, assume:

1) The sample is a simple random sample.

(In a simple random sample, each sample of size n has exactly the same chance of being selected. Can't be stratified, cluster, or convenience.)

2) The distribution is a binomial distribution.

3) We can look at the binomial distribution as a normal distribution b/c $np \geq 5$ and $nq \geq 5$. This means we can use the Central Limit Thm.

Assumptions

1. SRS
2. Binomial
3. $np \geq 5$
   $nq \geq 5$

******Different results from samples are caused by chance random fluctuations -- not bad sampling techniques.

$\hat{p}$ can vary from sample to sample. But, p is based on the population-- it won't vary.

$\hat{p}$: point estimate (from survey)

p : actual value (unknown)

Often we can find the best point estimate of the population proportion, $\hat{p}$, but we don't know how good the estimate is.

SO WE NEED TO KNOW THE CONFIDENCE INTERVAL (CI) or INTERVAL ESTIMATE.

CI = a range of values used to estimate the true value of a population parameter.

CI is often associated with a confidence level such as 95%.

The confidence level helps find the CI.

CI: $.672 \pm .045$

write in the form $(\hat{p}-E, \hat{p}+E)$

$$(.627, .717)$$

$$\hat{p}-E < p < \hat{p}+E$$

$$.627 < p < .717$$

$.887 < p < .927$

Find $\hat{p}$ and E. $\Longrightarrow$ $.907 \pm .02$

$$E = \frac{\text{High} - \text{low}}{2}$$

$$E = \frac{.927 - .887}{2}$$

$$E = .02$$

$$\hat{p} = \frac{\text{High} + \text{low}}{2}$$

$$\hat{p} = \frac{.927 + .887}{2}$$

$$\hat{p} = .907$$

$.632 < p < .678$

Re-write in the form $\hat{p} \pm E$

$\hat{p} = .655 \quad E = .023$

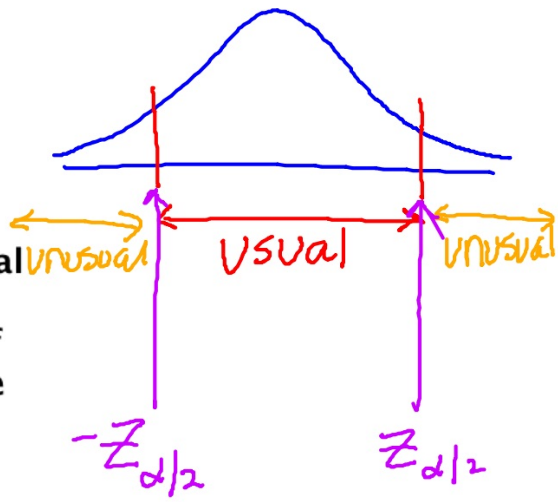
$.655 \pm .023$

p.312
5-10 All

Definition

Critical Value

- A **critical value** is the number on the borderline separating sample statistics that are likely to occur from those that are unlikely to occur. The number $z_{\alpha/2}$ is a critical value that is a z score with the property that it separates an area of $\alpha/2$ in the right tail of the standard normal distribution. (See Figure 6-2).

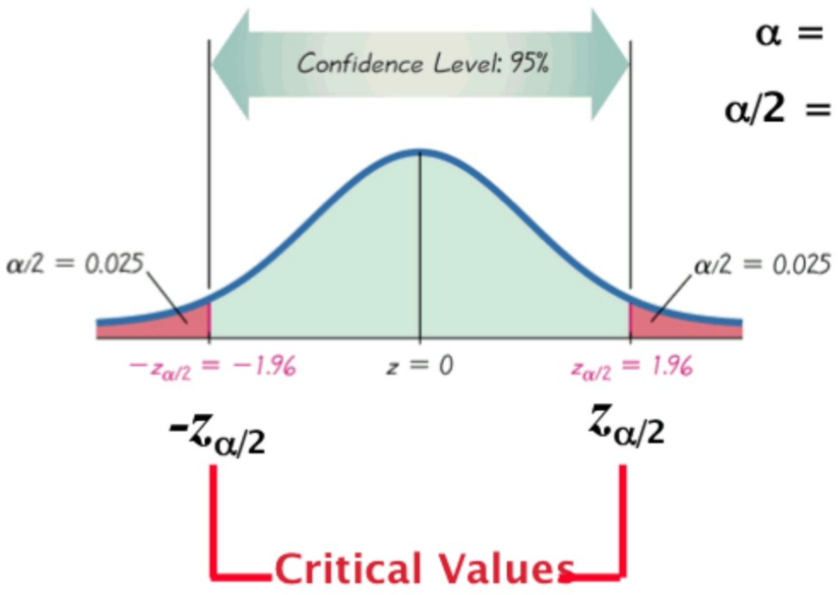


α : level of significance

Finding $z_{\alpha/2}$ for 95% Degree of Confidence

α = 5%

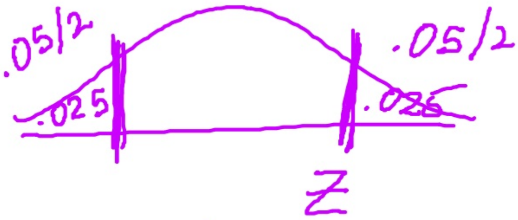
$\alpha/2$ = 2.5% = .025



Finding critical values given the level of confidence.

1) 95%

$\alpha = 1 - .95 = .05$



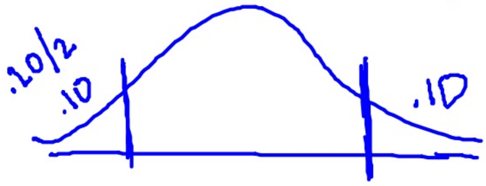
invnorm(.025)

$= -1.96$

$\boxed{Z_{\alpha/2} = 1.96}$

2) 80%

$\alpha = .20$



invnorm(.20/2)

$\boxed{Z_{\alpha/2} = 1.28}$

Finding critical values

3) 96%

$\alpha = .04$

invnorm$(.04/2)$

$Z_{\alpha/2} = 2.05$

4) 97%

$\alpha = .03$

invnorm$(.03/2)$

$Z_{\alpha/2} = 2.17$

The difference between the sample proportion $\hat{p}$ and the population proportion p is called the MARGIN OF ERROR.

Margin of Error of the Estimate of *p*

Formula 6-1

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$\hat{p}$: sample proportion

$\hat{q} = 1 - \hat{p}$

n : sample size

Find the margin of error.

$\hat{p} = \frac{x}{n}$

5) n = 200, x = 175, 95% confidence

$z_{\alpha/2} = 1.96$

$\hat{p} = .875$

$\hat{q} = .125$

$n = 200$

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$E = 1.96\sqrt{\frac{.875 \cdot .125}{200}}$$

$$E = .0458$$

Find the margin of error.

6) n = 500, 32% successes 94% confidence

$\hat{p} = .32$ $z_{\alpha/2} = 1.88$

$$E = 1.88\sqrt{\frac{.32 \cdot .68}{500}}$$

$$E = .0392$$

Find the margin of error.

7) n = 127, x = 94   90% confidence $z_{\alpha/2} = 1.64$

$\hat{p} = \frac{94}{127}$

127−94

$\hat{q} = \frac{33}{127}$

$E = 1.64\sqrt{\frac{94/127 \cdot 33/127}{127}}$

$1.645\sqrt{\frac{94/127 \cdot 33/127}{127}}$

$E = .0638$

$.0640$