5.3 Applications of Normal Distributions (Finding Values)

Things to keep in mind:

Probabilities and areas cannot be negative.

Z-scores to the left of the mean are negative. Z-scores to the right of the mean are positive.

Draw a graph to help you visualize!

Given a probability, we can find the corresponding z-score. But, if the distribution is not standard, you need to change the z-score back into a value that is relevant. ie use the mean, std dev, and z-score to find the value of x in the formula below

$$z = \frac{x - \text{mean}}{\text{std dev}}$$

$$x = z(\text{std dev}) + \text{mean}$$

$$Z = \frac{(X) - \mu}{\sigma}$$

$$Z\sigma = X - \mu$$

$$\boxed{Z\sigma + \mu = X}$$

IQ scores are normally distributed with a mean of 100 and a standard deviation of 15.

Find the x-score that corresponds to a z-score of 1.45.

$\mu = 100$

$\sigma = 15$

$z = 1.45$

$$z = \frac{x - \mu}{\sigma}$$

$$1.45 = \frac{x - 100}{15}$$

$$1.45 \cdot 15 = x - 100$$

$$1.45 \cdot 15 + 100 = x$$

$$\boxed{x = 121.75}$$

Finding Values

1) Find the z-score that corresponds to the given percentile or percentage.

2) Use the z-score formula to find x.

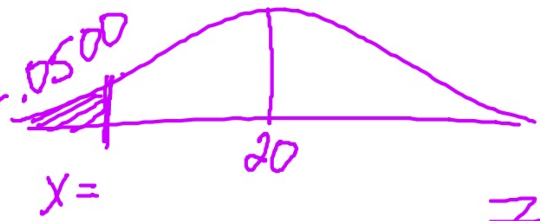
The weights of cereal boxes are normally distributed with a mean weight of 20 oz and a standard deviation of .07 oz.

Boxes in the lower 5% do not meet the minimum weight requirements. What is the minimum weight requirement for a cereal box?

$\mu = 20$

$\sigma = .07$

$X = \_\_\_\_$



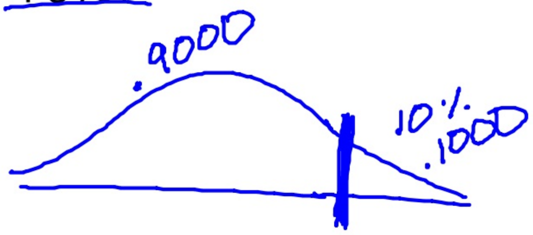
$Z = -1.645$

$$Z = \frac{X-\mu}{\sigma}$$

$$-1.645 = \frac{X-20}{.07}$$

$$\boxed{19.88\text{oz} = X}$$

Scores for a civil service exam are normally distributed with a mean of 75 and a standard deviation of 6.5. What is the lowest score you can receive and be in the top 10%?



$Z = 1.28$

look up .1000 on chart:

or

look up .9000 on chart

but know the z-score is positive

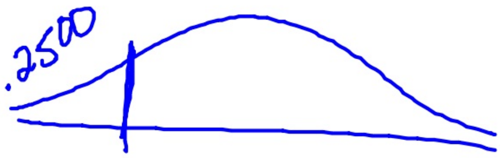
$$Z = \frac{X - \mu}{\sigma}$$

$$1.28 = \frac{X - 75}{6.5}$$

$$83.32 = X$$

Heights of men are normally distributed with a mean of 69.6 in and a standard deviation of 2.71 in.

What height represents the 25th percentile?



$Z = -.67$

$$-.67 = \frac{X - 69.6}{2.71}$$

$$67.78\text{in} = X$$

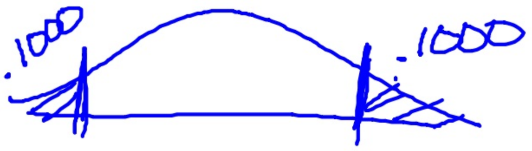
Assume that heights of women are normally distributed with a mean of 64.9 in and a standard deviation of 1.6 in.

If the top 10% and bottom 10% are excluded for an experiment, what are the cutoff heights to be eligible for this experiment?

cutoff heights

62.85 in

66.95 in



$Z = -1.28$

$\downarrow$

$-1.28 = \frac{X - 64.9}{1.6}$

$6285 = X$

$Z = 1.28$

$\downarrow$

$1.28 = \frac{X - 64.9}{1.6}$

$66.95 = X$