4.3: Binomial Distributions

In a **binomial experiment** there are **two** mutually exclusive outcomes, often referred to as "success" and "failure". If the probability of success is p, the probability of failure is $1 - p$.

Such an experiment whose outcome is random and can be either of two possibilities, "success" or "failure", is called a **Bernoulli trial**, after Swiss mathematician Jacob Bernoulli (1654 - 1705).

ples of Bernoulli trials:

ipping a coin -- heads is success, tails is failure

olling a die -- 3 is success, anything else is failure

oting -- votes for candidate A is success, anything else is failure

etermining eye color -- green eyes is success, anything else is failure

praying crops -- the insects are killed is success, anything else is fail

The probability of an event, p, occurring **exactly** r times:

$$P(r) = {}_nC_r \cdot p^r \cdot q^{n-r}$$

n = number of trials
r = number of specific events you wish to
    obtain
p = probability that the event will occur ← success
q = probability that the event will **not** ← failure
occur
    ($q = 1 - p$, the *complement* of the event)

test consists of 10 multiple choice questions with <u>five choices for</u>
uestion. As an experiment, you GUESS on each and every answer
it even reading the questions.

is the probability of getting <u>exactly</u> 6 questions correct on this test?

$$n = 10$$

$$r = 6$$

$$p = \frac{1}{5}$$

$$q = \frac{4}{5}$$

$$P(6) = {}_{10}C_6 \left(\frac{1}{5}\right)^6 \left(\frac{4}{5}\right)^{10-6}$$

$$= .00551$$

When rolling a die 100 times, what is the probability of rolling a "4" by 25 times?
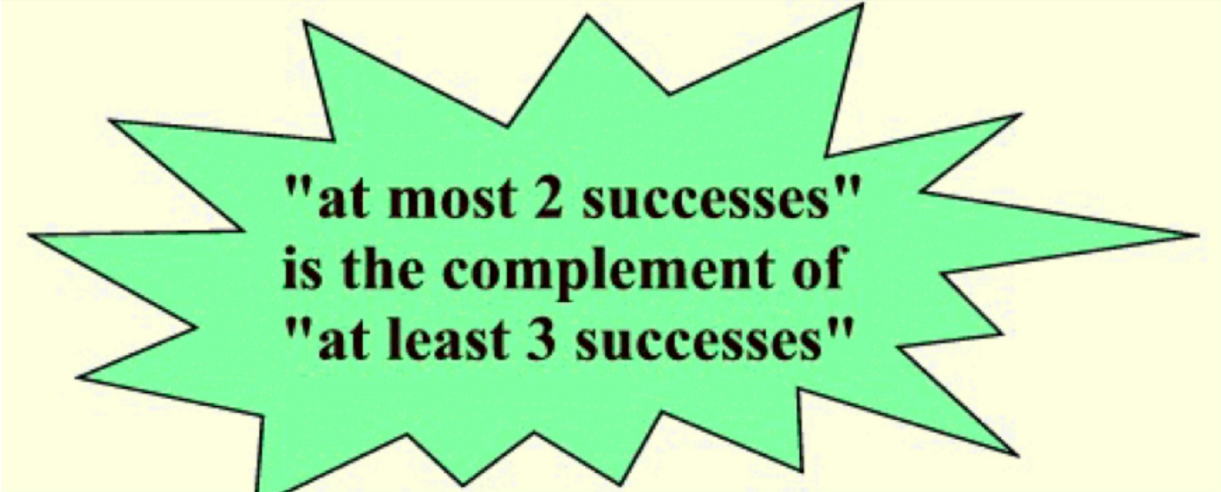
$$n = 100$$
$$r = 25$$
$$P = \frac{1}{6}$$
$$q = \frac{5}{6}$$

$$P(r) = {}_nC_r \, (p)^r \, (q)^{n-r}$$

$$P(25) = {}_{100}C_{25} \left(\frac{1}{6}\right)^{25} \left(\frac{5}{6}\right)^{75}$$

$$= .00983$$

http://www.regentsprep.org/Regents/math/algtrig/ATS7/B
Prac.htm

"at most 2 successes"
is the complement of
"at least 3 successes"

$n = 10 \qquad P = 1/4 \qquad q = 3/4$

3. There are 10 questions on a multiple choice quiz with 4 choices each. What is the probability of getting 3 or 4 questions correct?

$$P(3 \text{ or } 4) = P(3) + P(4)$$

$$= {}_{10}C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^7 \qquad + {}_{10}C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^6$$

$$= .396$$

4. In a school exam, one student out of 10 failed the test. Determine the probability that out of 20 students picked at random, at most 2 of them failed.

$n = 20$

$P(0 \text{ or } 1 \text{ or } 2) = P(0) + P(1) + P(2)$

$P = \frac{1}{10}$

$q = \frac{9}{10}$

$$= {}_{20}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20} +$$

$$20 \, C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{19} +$$

$$20 \, C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{18} = .677$$