3.7 Counting (sounds easy, doesn't it?)

1

How many outfits can you make out of 3 different shirts, 4 pairs of pants, and 2 pairs of shoes?

$3 \cdot 4 \cdot 2 = 24$

Fundamental Counting Rule

For a sequence of 2 events in which the first event can occur a *ways and the second event can occur* b *ways, the events together can occur a total of* ab *ways.*

How many even 2 digit positive integers less than 50 are there?

$4 \cdot 5$

20

2

How many different ways can you order a cone of ice cream if you have 10 flavors to choose from, 3 different size cones, and 4 different toppings? (You must choose a topping.)

$10 \cdot 3 \cdot 4$

120

(3) How many different passwords can you make if the password must be 6 characters long and the first 5 characters must be letters and the last character must be a digit?

$26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 10$

$26^5 \cdot 10$

(4) How many different arrangements can be made from the letters in the word "math" if none of letters can be repeated?

MATH

$4 \cdot 3 \cdot 2 \cdot 1$

$4!$

What if they letters can be repeated?

$4 \cdot 4 \cdot 4 \cdot 4$

4^4

6 x 5 x 4 x 3 x 2 x1 can be represented as 6!
(Read 6 factorial)

$0! = 1$

N! = N(N-1)(N-2)(N-3)....1

Look on your calculator under MATH and PRB and you'll see the !.

**** FACTORIAL RULE ****

A collection of n different items can be arranged in order n! different ways. (This does not allow for repetition of items.)

5 books arrange on a shelf.

$5!$ $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

5

How many different routes are there to go through to 12 classrooms in the building?

$12!$ 479,001,600

$12P12$

(6)

How many different ways can you arrange all the 26 letters in the alphabet?

$26!$

7

How many different ways can you arrange the letters in the alphabet if you only select 6 letters?

AB is not the same as BA.

permutation

$26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21$

$_{26}P_{6}$

order matters

Permutations (when items are different)

$_{26}P_6$

the number of permutations (sequences/ arrangements) of r items selected from n available items (without replacement) is

$\frac{n!}{(n-r)!}$ $\frac{26!}{20!}$

$$_nP_r = \frac{n!}{(n-r)!}$$

Look under MATH PRB for this function.

$26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot \cancel{20 \cdot 19 \cdot 18 \cdots}$

Rearrangements of the same items are different sequences ie ABC is different than BCA in a permutation.

(8) Ariela wants me to paint 5 different color stripes in her room. How many different sequences of 5 colors can she choose from yellow, green, blue, light blue, pink, orange, purple, red, grey, white, and black?

11 colors

choose 5

$_{11}P_5$

Permutation: order matters

Permutations Rule when some items are identical

If there are n items with n_1 alike, n_2 alike...the number of permutations of all n items is

$$\frac{n!}{n_1!n_2!\ldots n_k!}$$

$$\frac{n!}{n_1!n_2!\ldots n_k!}$$

$6!/(3!2!)$

BANANA

$$\frac{6!}{3!2!} = \frac{6!}{12} = 60$$

9

How many ways can you arrange the letters in the word "hippopotamus"?

$$\frac{12!}{3!2!}$$

$$\frac{12!}{12}$$

$$\frac{27!}{9!} \neq 3!$$

10

How many different ways can you choose 3 students from a class of 30? (Note: Does the order they are chosen make a difference? NO)

order does not matter

combination

$_{30}C_3$ 4060

#10

Combinations Rule (without replacement)

The number of combinations of r items selected from n *different* items is ${}_nC_r = \frac{n!}{(n-r)!r!}$

$${}_{30}C_3 = \frac{30!}{(27! \cdot 3!)} = 4060$$

Use the permutation rule when different orderings of the same items are counted separately. Use the combination rule when different orderings of the same items are not counted separately.

In a permutation ABC is not considered the same as BCA.

In a combination ABC is considered the same as BCA.

HOW DO YOU KNOW WHEN TO USE WHICH RULE? THINK!!! DOES THE ORDER MATTER? IF NOT USE, COMBINATIONS.

11

Mr. Neely has a group of 10 students that he meets with monthly. He wants a 3-person committee to chair the meetings: a chairperson, gopher, and secretary. (WHICH QUESTION IS A COMBINATION AND WHICH IS A PERMUTATION?)

a) How many different 3-person committees are possible?

Combination ${}_{10}C_3$

b) How many different ways can the positions be filled?

Permutation ${}_{10}P_3 = 10 \cdot 9 \cdot 8$

Mrs. Smith
Alg. 2

30

randomly
select

5

$_{30}C_5 = 142,506$

$P(\text{top 5 students}) = \frac{1}{142,506}$

$= .00000702$

12

What is the probability that you will win the FL lottery if you buy one ticket? Remember, you have to choose all 6 of the winning numbers. The numbers are from 1-53. (IS THIS A PERMUTATION OR A COMBINATION?)

$$P(\text{win}) = \frac{1}{{}_{53}C_6} = .0000000436$$

13

Find the probability of winning the NY lottery if you have to select the 5 winning numbers from 1-39.

$$P(\text{win}) = \frac{1}{{}_{39}C_5} = .00000174$$

14 *If a combination lock has the numbers 0 - 29 and requires 3 numbers to open the lock, how many different 3 number sequences can be chosen?*

order matters : permutation

$30 \cdot 30 \cdot 30$

$30^3 = 27{,}000$

12 - 18 - 12

$P(\text{open}) = \frac{1}{27{,}000}$

$= 0.00000370$