5.5 THE CENTRAL LIMIT THEOREM

Things to remember,
random variable-a variable with a single numerical value that is determined by chance for each trial

probability distribution- a graph, table etc that gives the probability for each each value of the random variable

sampling distribution of the mean- the probability distribution of sample means (each sample is the same size) (NO WE DID NOT STUDY THE SECTION ON THIS)

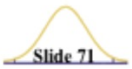
As the sample size increases, the corresponding sample means vary less.

The Central Limit Theorem

$n > 30$

If the sample size is large enough, the distribution of sample means can be approxiamated by a normal distribution, even if the original population is not normally distributed.

Practical Rules Commonly Used:



1. For samples of size n larger than 30, the distribution of the sample means can be approximated reasonably well by a normal distribution. The approximation gets better as the sample size n becomes larger.

2. If the original population is itself normally distributed, then the sample means will be normally distributed for any sample size n (not just the values of n larger than 30).

Notation

the mean of the sample means

$$\mu_{\bar{x}} = \mu$$

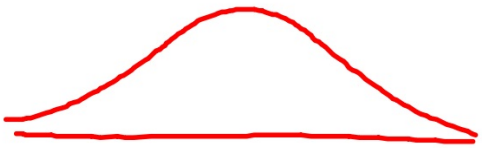
the standard deviation of sample mean

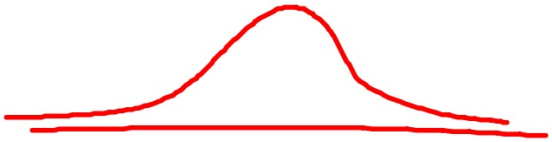
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

(often called standard error of the mean)

As the sample size increases, the sampling distribution of sample means approaches a normal distribution.

Means vary less





less spread

smaller st. dev.

OKAY...so how do you use the Central Limit Theorem?

If you are using an individual value from a normally distributed population, find the z score and go from there–just like you did in 5.3.

$$z = \frac{x - \mu}{\sigma}$$

If you are working with a mean for some sample use .

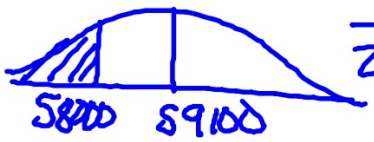
$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

The population mean annual salary for registered nurses is normally distributed with a mean of $59,100 and a standard deviation of $1700.

What is the probability of randomly selecting a registered nurse whose salary is less than $58,000

$Z = \frac{58000 - 59100}{1700} = -.65$ $P(x < 58000) = .2578$

If a random sample of 25 nurses is selected, find the probability that the mean annual salary of the sample is less than $58,000?



$Z = \frac{(58000 - 59100)}{\left(\frac{1700}{\sqrt{25}}\right)} = -3.24$

.0006

$P(\bar{x} < 58000) = .0006$

Notice in the previous question that it is less likely that 25 randomly selected nurses will have an income less than $58,000.

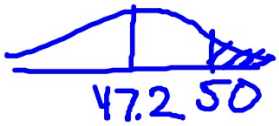
In other words, the means of the salaries vary less than the individual.

If the sample size is large enough ($n > 30$) then the sample means will be normally distributed (regardless of the distribution of the population.)

The mean age of employees at a large corporation is 47.2 years with a standard deviation of 5.6 years. Random samples of size 32 are drawn from this population.

ncdf (2.83, 888)

Find the probability that the mean age of the sample is more than 50 years?



$$Z = \frac{50 - 47.2}{5.6/\sqrt{32}} = 2.83$$

Why doesn't it matter that the question doesn't specify that the distribution is normal?

$P(\bar{x} > 50) = .0023$

The Boston Women's club needs an elevator limited to 8 passengers. The club has 120 women members with weights that approximate a normal distribution mean with a mean of 143 lb and a standard deviation of 29 lb.

a) If 8 different women are randomly selected, find the probability that their total weight will not exceed the maximum capacity of 1300 lbs.

b) If we want a .99 probability that the elevator will not be overloaded whenever 8 people are randomly selected as passengers, what should the maximum allowable weight be?

You need to build a bench that will seat 18 male college football players and you must first determine the length of the bench. Men have hip breadths that are normally distributed with a mean of 14.4 in. and a standard deviation of 1.0 in.

a) What is the minimum length of the bench if you want a 0.975 probability that it will fit the combined hip breadths of 18 randomly selected men?

b) What would be wrong with actually using the result from part(a) as the bench length?