

Solving Polynomial Equations by Factoring

I.

1. What is a polynomial?

II.

a) How many solutions does the equation have?

b) Solve by *factoring*. Make sure to state the multiplicity of any repeated solutions.

c) Describe the solutions: How many solutions are real? How many solutions are imaginary?

2. $x(x-1)^2(x+1)^3 = 0$

3. $x^3 - 2x^2 + x = 0$

4. $12x^3 + 4x^2 + 3x + 1 = 0$

5. $x^5 + 2x^4 + 3x^2 - 12 = x^5 + x^4 + 2x^2$

6. $x^3 - 8 = 0$

7. $(2x-1)(x^2+3)(2x^2-5) = 0$

8. $y^4 - 1 = 0$

9. $5x^6 = 9x^4 - 4x^2$

10. $27x^3 = -1$

11. $15x^5 - 72x^3 - 108x = 0$

12. $x^6 - 4x^4 - 9x^2 + 36 = 0$

13. $4x^4 + 4x^2 = -1$

14. $x^7 - x^4 = 0$

15. $18x^3 = 50x$

16. $6x^2 - 17x - 45 = 0$

17. $x^6 - 64 = 0$

III. Write a polynomial equation in standard form with integral coefficients with the given roots.

18. 1, 3, $-1/2$

19. 0, 2 (multiplicity of 2)

20. $\sqrt{2}$, $-\sqrt{2}$, 4

ANSWERS

1. A polynomial is an algebraic expression whose term(s) have variables with whole exponents, real coefficients, and there is no division by variables.
2.
 - a. 6
 - b. $x = 0$, 1 mult. 2, -1 mult. 3
 - c. 6 real, no imaginary
3.
 - a. 3
 - b. $x = 0$, 1 mult. 2
 - c. 3 real, no imaginary
4.
 - a. 3
 - b. $x = -\frac{1}{3}, \pm\frac{1}{2}i$
 - c. 1 real, 2 imaginary
5.
 - a. 4
 - b. $x = \pm\sqrt{3}, \pm 2i$
 - c. 2 real, 2 imaginary
6.
 - a. 3
 - b. $x = 2, -1 \pm i\sqrt{3}$
 - c. 1 real, 2 imaginary
7.
 - a. 5
 - b. $x = \frac{1}{2}, \pm i\sqrt{3}, \pm \frac{\sqrt{10}}{2}$
 - c. 3 real, 2 imaginary
8.
 - a. 4
 - b. $x = \pm 1, \pm i$
 - c. 2 real, 2 imaginary
9.
 - a. 6
 - b. $x = 0$ mult 2, $\pm 1, \pm \frac{2\sqrt{5}}{5}$
 - c. 6 real, no imaginary
10.
 - a. 3
 - b. $x = -\frac{1}{3}, \frac{1}{6} \pm \frac{\sqrt{3}}{6}i$
11.
 - a. 5
 - b. $x = 0, \pm\sqrt{6}, \pm \frac{i\sqrt{30}}{5}$
 - c. 3 real, 2 imaginary
12.
 - a. 6
 - b. $x = \pm i\sqrt{3}, \pm\sqrt{3}, \pm 2$
 - c. 4 real, 2 imaginary
13.
 - a. 4
 - b. $x = \frac{i\sqrt{2}}{2}$ mult. 2, $-\frac{i\sqrt{2}}{2}$ mult. 2
 - c. 4 imaginary
14.
 - a. 7
 - b. $x = 0$ mult. 4, 1, $\frac{-1 \pm i\sqrt{3}}{2}$
 - c. 5 real, 2 imaginary
15.
 - a. 3
 - b. $x = 0, \pm \frac{5}{3}$
 - c. 5 real, 2 imaginary
16.
 - a. 2
 - b. $x = -\frac{5}{3}, \frac{9}{2}$
 - c. 2 real
17.
 - a. 6
 - b. $x = \pm 2, -1 \pm i\sqrt{3}, 1 \pm i\sqrt{3}$
 - c. 2 real, 4 imaginary
18. $2x^3 - 7x^2 + 2x + 3 = 0$
19. $x^3 - 4x^2 + 4x = 0$
20. $x^3 - 4x^2 - 2x + 8 = 0$