

Riemann Sum and Trapezoidal Approximations Worksheet – 2

1. $\int_1^9 \frac{1}{x} dx, n=4$

- Estimate the value of the integral using a Right Riemann Sum. Is this approximation an over or under approximation? JYA.
- Estimate the value of the integral using a Left Riemann Sum. Is this approximation an over or under approximation? JYA.
- Estimate the value of the integral using a Trapezoidal Approximation. Is this approximation an over or under approximation? JYA.

2. $\int_{-\frac{\pi}{2}}^0 \cos x dx, n=2$

- Estimate the value of the integral using a Right Riemann Sum. Is this approximation an over or under approximation? JYA.
- Estimate the value of the integral using a Left Riemann Sum. Is this approximation an over or under approximation? JYA.
- Estimate the value of the integral using a Trapezoidal Approximation. Is this approximation an over or under approximation? JYA.

3. Use a trapezoidal approximation with 4 subintervals indicated by the data below to approximate:

$$\int_0^{24} f(x) dx$$

x	0	8	14	22	24
f(x)	0	7	-8	2	0

4. Use a Midpoint Riemann Sum with 4 subintervals indicated by the data below to approximate:

$$\int_0^{80} f(x) dx$$

x	0	10	20	30	40	50	60	70	80
f(x)	5	14	22	29	35	40	44	47	49

5. Use a Midpoint Riemann Sum with 4 subintervals indicated by the data below to approximate:

$$\int_0^{40} f(x) dx$$

x	0	5	10	15	20	25	30	35	40
f(x)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

6. Use a right Riemann sum with subintervals indicated by the data below to approximate:

$$\int_0^{12} f(x) dx$$

x	0	2	5	7	11	12
f(x)	5.7	4.0	2.0	1.2	0.6	0.5

7. Use a trapezoidal approximation with subintervals indicated by the data below to approximate:

$$\int_0^{360} f(x) dx$$

x	0	60	120	180	240	300	360
f(x)	24	30	28	30	26	24	26

ANSWERS

1.

a. $2\left[\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9}\right] = 1.575$, This approximation is an under estimate because $f(x) = \frac{1}{x}$ is

decreasing on the interval $(1, 9)$.

b. $2\left[1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7}\right] = 3.352$, This approximation is an over estimate because $f(x) = \frac{1}{x}$ is

decreasing on the interval $(1, 9)$.

c. $\left(\frac{1}{2}\right)(2)\left[1 + 2\left(\frac{1}{3}\right) + 2\left(\frac{1}{5}\right) + 2\left(\frac{1}{7}\right) + \frac{1}{9}\right] = 2.463$, This approximation is an over estimate

because $f(x) = \frac{1}{x}$ is concave up on the interval $(1, 9)$.

2.

a. $\frac{\pi}{4}\left[\cos\left(-\frac{\pi}{4}\right) + \cos(0)\right] = 1.341$, This approximation is an over estimate because

$f(x) = \cos x$ is increasing on the interval $\left(-\frac{\pi}{2}, 0\right)$.

b. $\frac{\pi}{4}\left[\cos\left(-\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{4}\right)\right] = 0.555$, This approximation is an under estimate because

$f(x) = \cos x$ is increasing on the interval $\left(-\frac{\pi}{2}, 0\right)$.

c. $\left(\frac{1}{2}\right)\left(\frac{\pi}{4}\right)\left[\cos\left(-\frac{\pi}{2}\right) + 2\cos\left(-\frac{\pi}{4}\right) + \cos(0)\right] = 0.948$, This approximation is an under

estimate because $f(x) = \cos x$ is concave down on the interval $\left(-\frac{\pi}{2}, 0\right)$.

3. $\frac{1}{2}(8)(0+7) + \frac{1}{2}(6)(7-8) + \frac{1}{2}(8)(-8+2) + \frac{1}{2}(2)(2+0) = 115$

4. $20(14 + 29 + 40 + 47) = 2600$

5. $10(9.2 + 7.0 + 2.4 + 4.3) = 229$

6. $(2)(4) + (3)(2) + (2)(1.2) + (4)(0.6) + (1)(0.5) = 19.3$

7. $\left(\frac{1}{2}\right)(60)\left[24 + 2(30) + 2(28) + 2(30) + 2(26) + 2(24) + 26\right] = 9780$