Riemann Approximations Worksheet 1

I.
For each question below:

a) Estimate the value of the integral using a Right Riemann Sum.
b) Estimate the value of the integral using a Left Riemann Sum.
c) Estimate the value of the integral using a Midpoint Riemann Sum.
d) Find the exact value of the integral using your calculator, then state whether the answers to parts a & b are over or underapproximations.

Let *n* equal the number of rectangles.

1. $\int_0^2 x^3 dx$, $n = 2$

2. $\int_1^9 \sqrt{x} dx$, $n = 4$

3. $\int_1^2 \frac{1}{x^2} dx$, $n = 4$

II.
4. When are Right Riemann Sums overapproximations/underapproximations?
5. When are Left Riemann Sums underapproximations/underapproximations?

ANSWERS

1.

a) $\int_0^2 x^3 dx \approx (1)(1^3 + 2^3) = 9$

b) $\int_0^2 x^3 dx \approx (1)(0^3 + 1^3) = 1$

c) $\int_0^2 x^3 dx \approx (1)\left(\left(\frac{1}{2}\right)^3 + \left(\frac{3}{2}\right)^3\right) = \frac{7}{2}$

d) $\int_0^2 x^3 dx = 4$, a) over, b) under

2.

a) $\int_1^9 \sqrt{x} dx \approx (2)\left(\sqrt{3} + \sqrt{5} + \sqrt{7} + \sqrt{9}\right) \approx 19.228$

b) $\int_1^9 \sqrt{x} dx \approx (2)\left(\sqrt{1} + \sqrt{3} + \sqrt{5} + \sqrt{7}\right) \approx 15.228$

c) $\int_1^9 \sqrt{x} dx \approx (2)\left(\sqrt{2} + \sqrt{4} + \sqrt{6} + \sqrt{8}\right) \approx 17.384$

d) $\int_1^9 \sqrt{x} dx = \frac{52}{3}$, a) over, b) under

3.

a) $\int_1^2 \frac{1}{x^2} dx \approx \left(\frac{1}{4}\right)\left(\left(\frac{4}{5}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{4}{7}\right)^2 + \left(\frac{1}{2}\right)^2\right) \approx .415$

b) $\int_1^2 \frac{1}{x^2} dx \approx \left(\frac{1}{4}\right)\left((1)^2 + \left(\frac{4}{5}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{4}{7}\right)^2\right) \approx .602$

c) $\int_1^2 \frac{1}{x^2} dx \approx \left(\frac{1}{4}\right)\left(\left(\frac{8}{9}\right)^2 + \left(\frac{8}{11}\right)^2 + \left(\frac{8}{13}\right)^2 + \left(\frac{8}{15}\right)^2\right) \approx .495$

d) $\int_1^2 \frac{1}{x^2} dx = \frac{1}{2}$, a) under, b) over

4. Answers will be discussed in class.
5. Answers will be discussed in class.