2.6 Related Rates

1. The length of a rectangle is decreasing at the rate of 2 cm/sec while the width is increasing at the rate of 2 cm/sec. When the length is 12 cm and the width is 5 cm, find the rates of change of:
   a) the area
   b) the perimeter
   c) the length of a diagonal of the rectangle

2. A 13 ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at a rate of 5 ft/sec.
   a) How fast is the top of the ladder sliding down the wall at that moment?
   b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing at that moment?
   c) At what rate is the angle between the ladder and the ground changing at that moment?

3. A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius, *r*, of the outer ripple is increasing at a constant rate of 1 ft/s. When the radius is 4 ft, at what rate is the total area *A* of the disturbed water changing?

4. Air is being pumped into a spherical balloon at a rate of 4.5 cubic ft per minute. Find the rate of change of the radius when the radius is 2 ft.

5. A hot air balloon rising straight up from a level field is tracked by a range finder 500 feet from the lift-off point. At the moment the range finder's elevation angle is $\frac{\pi}{4}$, the angle is increasing at the rate of 0.14 radians per minute. How fast is the balloon rising at that moment?

6. Find the rate of change of the distance between the origin and a moving point on the graph of y=sinx if *dx/dt* = 2 centimeters per second. (Your answer will be in terms of x!!!)

7. An airplane is flying at an altitude of 6 miles on a flight path that will take it directly over a radar tracking station. If the distance from the plane to the station, *s*, is decreasing at a rate of 400 miles per hour when *s* = 10, what is the speed of the plane?

8. A fish is reeled in at a rate of 1 ft per second from a point 10 feet above the water. At what rate is the angle between the line and the water changing when there is a total of 25 ft of line out?

9. A baseball diamond has the shape of a square with sides 90 feet long. A player running from second to third base at a speed of 28 ft/s is 30 ft from third base. At what rate is the player's distance from home plate changing?

10. A television camera at ground level is filming the lift off of a space shuttle that is rising vertically according to the position equation $s = 50t^2$, where *s* is measure in feet and *t* is measured in seconds. The camera is 2000 feet from the launching pad. Find the rate of change of the angle of elevation of the camera 10 seconds after lift-off.

11. A conical cup is 4 cm across and 6 cm deep. Water leaks out of the bottom at the rate of 2 cm^2/sec. How fast is the water level dropping when the height of the water is 3 cm?

12. Air is escaping from a spherical balloon at the rate of 2 cm^3/min. How fast is the surface area shrinking when the radius is 1 cm?

ANSWERS

1.
a. dA/dt = 14cm^2/sec
b. dP/dt = 0 cm/sec
c. dD/dt = -14/13 cm/sec

2
a. -12 ft/sec
b. -119/2 ft^2/s
c. -1 radian/sec

3. dA/dt = 8Π ft^2/sec

4. dr/dt = 0.09 ft/min

5. 140 ft/min

6. $\frac{dD}{dt} = \frac{2x + 2\sin x \cos x}{\sqrt{x^2 + \sin^2 x}}$

7. 500 mph

8. dΘ/dt = $\frac{2}{25\sqrt{21}}$ rad/sec

9. dH/dt = -8.85 ft/sec

10. dΘ/dt = 0.069 rad/sec

11. dh/dt = $-\frac{2}{\pi}$ cm/sec

12. dSA/dt = -4 cm^2/min