Rates of change and Rectilinear Motion

- Application of the Derivative: slope, tangent lines, rate

- Rate of Change(definition): change in y ÷ change in x

Examples:

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When you are asked to find the rate of change, you are finding the slope.

Find the rate of change of $f(t) = (t^2 + 1)^3$ at $t = 1$

$$f'(t) = 3(t^2+1)^2 \cdot 2t$$

$$f'(1) = 3(2)^2 \cdot 2$$

$$= 24$$

- Rectilinear Motion Problems – When we talk about these types of problems, we often talk about three types of functions

1. position:

ft Notation: $x(t)$, $s(t)$, $f(t)$

Application(s): find the position at $t=2$; find t when $s(t)=0$

2. velocity:

Notation: $x'(t)$, $s'(t)$, $v(t)$

ft/sec Application(s): find velocity at $t=3$

find velocity when the object hits the grand

3. acceleration:

Notation: $x''(t)$, $s''(t)$, $v'(t)$, $a(t)$

ft/sec^2 Application(s): find acceleration at $t=2$

Ex 3: At time t=0 seconds a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$.

a) When does the diver hit the water?

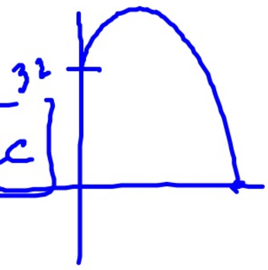
when $h = 0$ ft

$$0 = -16t^2 + 16t + 32$$

$$0 = -16(t^2 - t - 2)$$

$$-16(t-2)(t+1)$$

$$\boxed{t = 2\text{sec}}$$



b) What is the diver's velocity at impact?

$$s'(t) = -32t + 16$$

$$s'(2) = -64 + 16$$

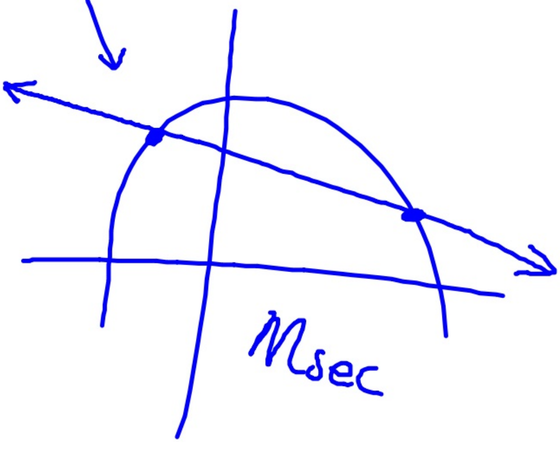
$$= -48 \text{ ft/sec}$$

- Average Velocity vs. Instantaneous Velocity
  - Average Velocity: given an interval, find the slope of the line connecting the endpoints (slope of secant)

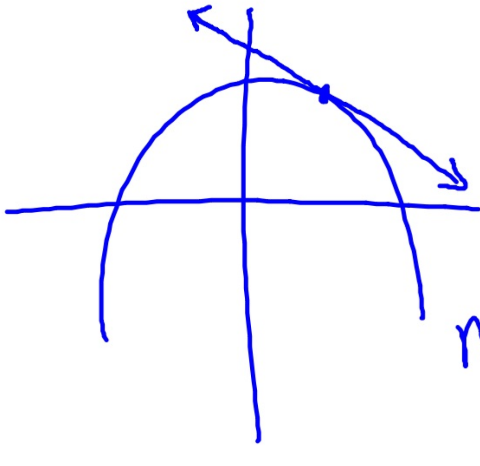
    Formula: $[a, b]$ $\dfrac{f(b)-f(a)}{b-a}$

  - Instantaneous Velocity: slope of tangent at a point

    Formula: $s'(c)$ or $v(c)$

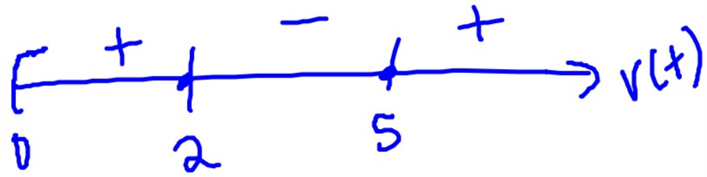


m_{sec}



m_{tan}

- Speed: $|v(t)|$
- Rest: $v(t) = 0$
- Left and Right Motion:
    - Left: $v(t) < 0$
    - Right: $v(t) > 0$
    - Changes Direction: $v(t) = 0$ and $v(t)$ changes from positive to negative **or** $v(t)$ changes from negative to positive



Ex 4: A billiard ball is dropped from a height of 100 ft, its height *s* at time *t* is given by the position function $s(t) = -16t^2 + 100$, where *s* is measured in feet and *t* is measured in seconds.

a) Find the average velocity over time interval [1, 2].

$(1, 84)$ $(2, 36)$

$$\frac{s(2) - s(1)}{2 - 1} = \frac{36 - 84}{2 - 1} = -48 \text{ ft/sec}$$

b) Find the Instantaneous velocity at the endpoints of the interval.

$t = 1$

$s'(t) = -32t$

$s'(1) = -32$ ft/sec

$t = 2$

$s'(2) = -64$ ft/sec

c) Find the speed at the endpoints of the interval.

speed
$t = 1$
32 ft/sec

speed
$t = 2$
64 ft/sec

Ex 5: A particle starts at time t=0 and moves along the x-axis so that its position at any time $\boxed{t \ge 0}$ is given by $x(t) = (t-1)^3(2t-3)$.

a) Find the velocity of the particle at any time $t \ge 0$. Simplify.

$v(t) = (t-1)^2(8t-11)$

b) Determine the values of t for which the particle is at rest.

$0 = (t-1)^2(8t-11)$

$t = 1, 11/8$

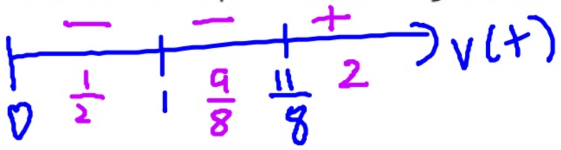
1 sec

11/8 sec

c) Determine the values of t for which the particle is moving to the left. JYA.

$(0, 1)$

$(1, \frac{11}{8})$



Moving to the left because v(t) < 0 on these intervals.

$$x(t) = (t-1)^3(2t-3)$$

$$x'(t) = (t-1)^3(2) + (2t-3)\left(3(t-1)^2 \cdot 1\right)$$

$$= (t-1)^2\left(2(t-1) + 3(2t-3)\right)$$

$$\boxed{x'(t) = (t-1)^2(8t-11)}$$

d) Determine the values of t for which the particle is moving to the right. JYA

$(11/8, \infty)$ because $v(t) > 0$ on this interval

e) Determine the values of t for which the particle changes direction. JYA.

$t = 11/8$; $v\left(\frac{11}{8}\right) = 0$ and $v(t)$ changes signs at $t = 11/8$