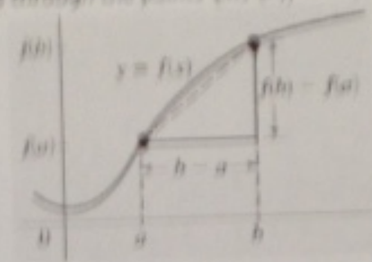
Average Rate of Change Notes

The average rate of change of the function $f(x)$ on the interval $x=a$ to $x=b$ is $[a,b]$

Average rate of change = $\frac{f(b)-f(a)}{b-a}$ $\frac{y_2-y_1}{x_2-x_1}$

The average rate of change is the *slope* of the secant line between $x=a$ and $x=b$ on the graph of $f(x)$, that is, the line that passes through the points $(a, f(a))$ and $(b, f(b))$.



Ex: Find the average rate of change on the indicated interval.

a) $f(x)=\frac{x-1}{x+2}$, $[0, 3]$

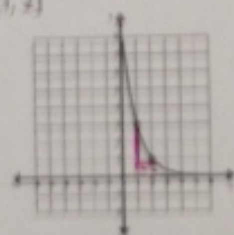
$(0, f(0)) \to (0, -\frac{1}{2})$

$(3, f(3)) \to (3, \frac{2}{5})$

$\frac{10\cdot\frac{2}{5}-\left(-\frac{1}{2}\right)\cdot 10}{10(3-0)} = \frac{9}{30}$

$= \frac{3}{10}$

b) $[1, 3]$



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c) $3<t<5$

Time (years)	1	2	3	4	5
Height(in.)	27	35	37	42	45

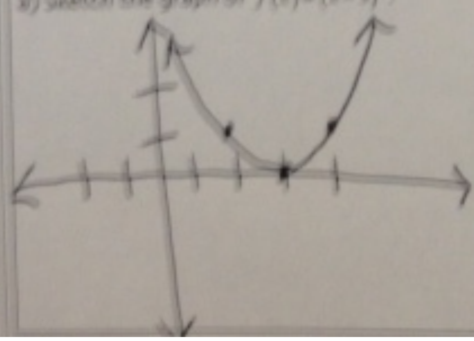
$\frac{45-37}{5-3} = 4$

or

$\frac{37-45}{3-5} = 4$ ✓

Ex:

a) Sketch the graph of $f(x)=(x-3)^2$.



Find the average rate of change of $f(x)=(x-3)^2$ on the indicated intervals.

b) $x=-1$ to $x=2$

$(-1, 16)$
$(2, 1)$

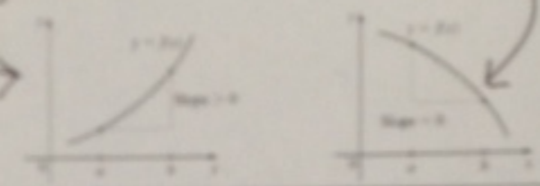
$\frac{16-1}{-1-2} = -5$

c) $[4, 7]$

$(4, 1)$
$(7, 16)$

$\frac{16-1}{7-4} = 5$

- If $f(x)$ is increasing on the interval [a, b], then the average rate of change of $f(x)$ is ____positive____ on the interval [a, b].
- If $f(x)$ is decreasing on the interval [a, b], then the average rate of change of $f(x)$ is ____negative____ on the interval [a, b].



Ex: On which interval does the function $f(x)=\frac{1}{8}x^3-x^2$ have an average rate of change of $\frac{1}{2}$?

$-2<x<2$
$0<x<4$ X
$-3<x<2$
$-4<x<1$ X

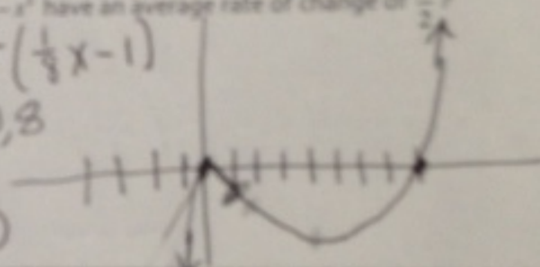
$(-2,-5)$

$(2,-3)$

$0=x^2\left(\frac{1}{8}x-1\right)$

$x=0,8$

$\frac{1}{2}=\frac{-3-(-5)}{2-(-2)}$



Ex: The graph below shows the United States population from 1900 to 2010, as recorded by the U.S. Census Bureau.

a) What was the rate of change in the population from 1900 to 2000? Is this greater or less than the rate of change in the population from 2000 to 2010?

(1900, 75) (2000, 275)

2 millions of people/year

(2000, 275) (2010, 320)

4.5 millions of people/year

b) Which 10-year time periods have the highest and the lowest rates of change? How did you find these?

2000-2010 1930-1940