# Rate In/ Rate Out AP Questions

1. Calculator

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12\sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time $t$, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

2. Calculator

A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \le t \le 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

(a) Is the amount of water in the tank increasing at time $t = 15$ ? Why or why not?

(b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$ ?

(c) At what time $t$, for $0 \le t \le 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.

(d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let $k$ be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of $k$.

3. Calculator

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time $t$ days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

4. Calculator

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where $t$ is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$ ? Why or why not?

(b) For what value of $t$ will the number of gallons of pollutant be at its minimum? Justify your answer.

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time $t$ does this model predict that the lake becomes safe?

5. No Calculator

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time $t$.

(d) At what time $t$, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

ANSWERS

1.
a) 70.571 gallons

b) Since H(6)-R(6)=-2.924<0, the heating oil is falling at t=6 hours.

c) 122.026 gallons

d) t=11.318 hours

2.
a) No, W(15)-R(15)=-121.09 <0

b) 1310 gallons

c) t=6.495 hours

d) $\int_0^k R(t)dt = 1310$

3.
a) Since R(6)=4.438>0, the number of mosquitoes is increasing at t=6.

b) Since $R'(6) = -1.913 < 0$, the number of mosquitoes is increasing at a decreasing rate at t=6.

c) 964 mosquitoes

d) 1039 mosquitoes

4.
a) Since $P'(9) = -0.646 < 0$, the amount of pollutant is not increasing at t=9.

b) P has a minimum at $t = (5\ln 3)^2$ days since P′ is negative on the interval $\left(0,\ (5\ln 3)^2\right)$ and positive on the interval $\left((5\ln 3)^2,\ \infty\right)$.

c) P(30.174)=35.104<40, so the lake is safe.

d) t=5 days

5.
a) 14/3

b) 148/3

c) $A(t) = 30 + \int_0^t \left(8 - \sqrt{x+1}\right)dx$

d) t=63 minutes