Quadratic Word Problems Notes

Falling objects

1) The height of a rocket launched upward from a 160-foot cliff is modeled by $h(t) = -16t^2 + 48t + 160$ where h(t) is the height in feet and t is the time in seconds.

a) What is the initial height of the rocket? What is the height of the rocket after 1 sec?

$h(0) = 160$ ft $\quad h(1) = 192$ ft

b) At what time does the rocket reach its maximum height?

$\frac{-b}{2a} = \frac{-48}{-32} = \frac{+3}{+2} = \frac{3}{2}$ sec

b) What is the maximum height?

$h\left(\frac{3}{2}\right) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 160$

$= -16\left(\frac{9}{4}\right) + 72 + 160 = -36 + 72 + 160 = 196$ ft

2) During a game of golf, Kayley hits her ball out of a sand trap. The height of the golf ball is modeled by the equation, $h = -16t^2 + 20t - 4$ where h is the height in feet and t is the time in seconds since the ball was hit. Find how long it takes Kayley's golf ball to hit the ground.

$0 = h$

$\frac{0}{4} = \frac{-16t^2}{4} + \frac{20t}{4} - \frac{4}{4}$

$0 = -4t^2 + 5t - 1$

$0 = 4t^2 - 5t + 1$

$0 = (4t - 1)(t - 1)$

$t = 1$ sec

3) The height of a flare fired from the deck of a ship in distress can be modeled by $h = -16t^2 + 112t + 56$ where h is the height of the flare above water in feet and t is the time in seconds. At what time(s) will the flare be at a height of 56 feet?

$56 = -16t^2 + 112t + 56$

$0 = -16t^2 + 112t$

$0 = -16t(t - 7)$

$t = 0, 7$ sec.

Numbers/Consecutive Integers

4) Find two positive consecutive odd integers such that the square of the first, added to 3 times the second, is 24.

$x^2 + 3(x + 2) = 24$

$x^2 + 3x - 18 = 0$

$(x + 6)(x - 3) = 0$

$x = 3, -6$

3, 5

5) The sum of the squares of two consecutive negative integers is 41. Find the integers.

$(-5)^2 + (-4)^2 = 41$

$x^2 + (x + 1)^2 = 41$

$x^2 + x^2 + 2x + 1 = 41$

$2x^2 + 2x - 40 = 0$

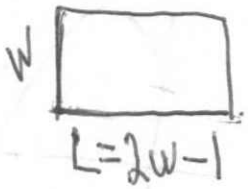
$2(x^2 + x - 20) = 0$

$2(x + 5)(x - 4) = 0$

$x = -5, -4$

Area

6) The length of a rectangle is 1 cm less than twice the width. The area is 45 cm^2. Find the dimensions of the photograph.



$W(2W-1)=45$

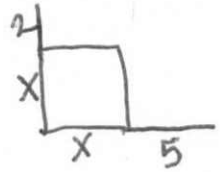
$2W^2-W-45=0$

$(2W+9)(W-5)=0$

$W=-\frac{9}{2}, 5$

5cm, 9cm

7) A square field has 5m added to its length and 2m added to its width. The field then had an area of 130 m^2. Find the length of a side of the original field.



$130=(X+5)(X+2)$

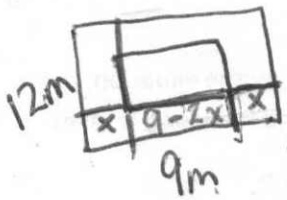
$130=X^2+7X+10$

$X^2+7X-120=0$

$(X+15)(X-8)=0$

$X=8m$

8) A decorator plans to place a rug in a 9 m by 12 m room so that a uniform strip of flooring around the rug will remain uncovered. How wide will this strip be if the area of the rug is to be half the area of the room?



108(Area room)

$(9-2x)(12-2x)=54$

$4x^2-42x+54=0$

$2(2x^2-21x+27)=0$

$2(2x-3)(x-9)=0$

$x=\frac{3}{2}$, ~~9~~

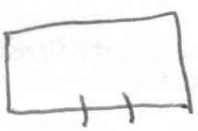
Misc

9) Suppose that one leg of a right triangle is 1 more than the other leg; and the hypotenuse is 1 less than 2 times the shorter leg. Find the lengths of all the sides.

$X^2+(X+1)^2=(2X-1)^2$

$X=3$

3, 4, 5



10) The profits for Mr. Unlucky's company can be modeled by the equation $P=-3t^2+18t-4$, where P is the amount of profit in thousands of dollars and t represents the number of years of operation. He realizes his company is on the downturn and wishes to sell before he ends up in debt.

a) When will Unlucky's business show a maximum profit?

$t=\frac{-b}{2a}=\frac{-18}{2(-3)}=3$ years

b) What is the maximum profit?

$P=-3(3)^2+18(3)-4=23$

$-27+54-4$

23 thousand dollars

c) At what time will it be too late to sell his business? (when will he start losing money?)

$0=-3t^2+18t-4$

$0=3t^2-18t+4$

$0=3(t^2-6t+9)-27+4$

$\frac{23}{3}=\frac{3(t-3)^2}{3}$

$\sqrt{\frac{69}{3}}=t-3$

$3+\sqrt{\frac{69}{3}}=t$