AP Style Polar Questions

1. (CALCULATOR) Let *S* be the region in the first quadrant bounded by the two graphs $y = \frac{2}{3}x$, $y = \sqrt{1 - \frac{x^2}{4}}$ and the *x*-axis The line and the curve intersect at point *P*.

a) Find the coordinates of *P*.
b) Set up and evaluate an integral expression that calculates the area of region *S*.
c) Find a polar equation to represent curve $y = \sqrt{1 - \frac{x^2}{4}}$.
d) Use the polar equation in (c) to set up and evaluate an integral expression that gives the area of the region *S*.

2. (CALCULATOR) Let $r = \theta + \cos(3\theta)$ for $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$, where *r* is measured in meters and θ is measured in radians.
a) Find the area bounded by the curve and the *y*-axis.
b) Find the angle θ that corresponds to the point on the curve with *y*-coordinate -1.
c) For what values of θ, $\pi \le \theta \le \frac{3\pi}{2}$ is $\frac{dr}{d\theta}$ positive? What does this say about *r*?

3. (CALCULATOR) Let *R* be the region bounded by $r = \frac{4}{1 + \sin\theta}$ for $0 \le \theta \le \pi$ and the *x*-axis.
a) Find the area of *R*.
b) Show the polar curve $r = \frac{4}{1 + \sin\theta}$ is $8y = 16 - x^2$ in rectangular form.

4.
Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

(A) $3\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$ (B) $3\int_0^{\pi} \cos^2\theta\, d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$ (D) $3\int_0^{\frac{\pi}{2}} \cos\theta\, d\theta$ (E) $3\int_0^{\pi} \cos\theta\, d\theta$

5.
(Calculator permitted) The area of the region enclosed by the polar graph of $r = \sqrt{3 + \cos\theta}$ is given by which integral?

(A) $\int_0^{2\pi} \sqrt{3 + \cos\theta}\, d\theta$ (B) $\int_0^{\pi} \sqrt{3 + \cos\theta}\, d\theta$ (C) $2\int_0^{\pi/2} (3 + \cos\theta)\, d\theta$

(D) $\int_0^{\pi} (3 + \cos\theta)\, d\theta$ (E) $\int_0^{\pi/2} \sqrt{3 + \cos\theta}\, d\theta$

6.

The area enclosed by one petal of the 3-petaled rose curve $r = 4\cos(3\theta)$ is given by which integral?

(A) $16\int_{-\pi/3}^{\pi/3} \cos(3\theta)\,d\theta$ (B) $8\int_{-\pi/6}^{\pi/6} \cos(3\theta)\,d\theta$ (C) $8\int_{-\pi/3}^{\pi/3} \cos^2(3\theta)\,d\theta$

(D) $16\int_{-\pi/6}^{\pi/6} \cos(3\theta)\,d\theta$ (E) $8\int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\,d\theta$

7.

If $a \neq 0$ and $\theta \neq 0$, all of the following must represent the same point in polar coordinates *except* which ordered pair?

(A) (a,θ) (B) $(-a,-\theta)$ (C) $(-a,\theta-\pi)$ (D) $(-a,\theta+\pi)$ (E) $(a,\theta-2\pi)$

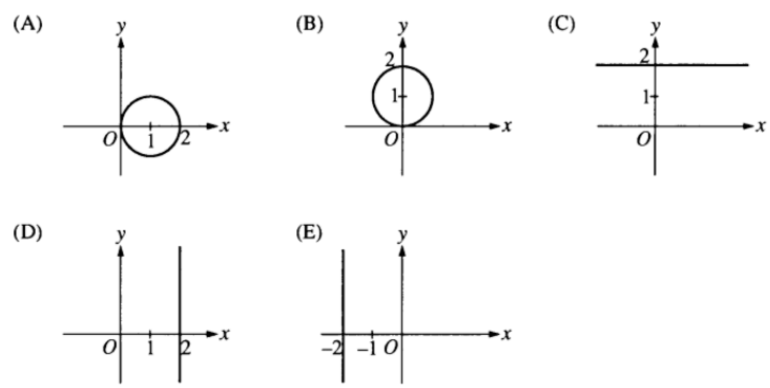
8.

Which of the following gives the slope of the polar curve $r = f(\theta)$ graphed in the *xy*-plane?

(A) $\dfrac{dr}{d\theta}$ (B) $\dfrac{dy}{d\theta}$ (C) $\dfrac{dx}{d\theta}$ (D) $\dfrac{dy/d\theta}{dx/d\theta}$ (E) $\dfrac{dy}{dx}\cdot\dfrac{dr}{d\theta}$

9.

Which of the following represents the graph of the polar curve $r = 2\sec\theta$?



10. (CALCULATOR) Find the length of the curve.

a) $r = 3\sin 2\theta$

b) $r = \cos\left(\dfrac{3\theta}{2}\right)$

11. Find the indicated area.

a) common interior of $r = 1+\cos\theta$ and $r = 1-\cos\theta$
b) common interior of $r^2 = \sin 2\theta$ and $r^2 = \cos 2\theta$
c) common interior of $r = a\sin\theta$ and $r = a\cos\theta$ if $a > 0$ and $b > 0$
d) inner loop of $r = 2-4\cos\theta$
e) between the loops of $r = 2-4\cos\theta$

ANSWERS

1.

a) $\left(\frac{6}{5},\frac{4}{5}\right)$

b) $A=\int_0^{6/5}\left(\frac{2}{3}x\right)dx+\int_{6/5}^{2}\left(\sqrt{1-\frac{x^2}{4}}\right)dx\approx 0.927$

c) $r^2=\frac{4}{\cos^2\theta+4\sin^2\theta}$ or $r=\frac{2}{\sqrt{\cos^2\theta+4\sin^2\theta}}$

d) $A=\frac{1}{2}\int_0^{\tan^{-1}(2/3)}\left(\frac{4}{\cos^2\theta+4\sin^2\theta}\right)d\theta\approx 0.927$

2.

a) $A=\frac{1}{2}\int_{\pi/2}^{3\pi/2}\left(\theta+\cos 3\theta\right)^2 d\theta\approx 19.675$

b) $\theta\approx 3.485$

c) $\frac{dr}{d\theta}>0$ for $(1.571,2.207)\cup(3.028,4.302)$. On these intervals the radius is increasing with respect to θ, thus the curve is moving *away* from the pole on these intervals.

3.

a) $A=\frac{1}{2}\int_0^{\pi}\left(\frac{4}{1+\sin\theta}\right)^2 d\theta\approx 10.667$

b)

$$r=\frac{4}{1+\sin\theta}$$
$$r=\frac{4}{1+y/r}$$
$$r=\frac{4r}{r+y}$$
$$r+y=4$$
$$r=4-y$$
$$r^2=16-8y+y^2$$
$$x^2+y^2=16-8y+y^2$$
$$x^2=16-8y$$

4. A
5. D
6. E
7. B
8. D
9. D

10.

a) 29.065
b) 15.865

11.

a) 0.712
b) 0.293
c) $\frac{a^2}{2}\left(\frac{\pi}{4}-\frac{1}{2}\right)$
d) 2.174
e) 33.351