Given $f(x) = x^2 + x - 3$ on $[0, 3]$, find c such that $f(c) = 3$.

$f(0) = -3$

$f(3) = 9$

$3 = x^2 + x - 3$

$0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

$x = \cancel{-3}, 2$

$c = 2$

Since $f(x)$ is continuous on $[0, 3]$ and $f(0) < 3 < f(3)$, IVT applies. There must exist a value c in $[0, 3]$, such that $f(c) = 3$.

$$g(x) = \begin{cases} cx + 3, & x \le 2 \\ x^2 + 4, & x > 2 \end{cases}$$

Find c so that g(x) is continuous everywhere.

$$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^+} g(x) = g(2)$$

$$\lim_{x \to 2^-} (cx+3) = \lim_{x \to 2^+} (x^2+4)$$

$$2c + 3 = 8 \qquad c = 5/2$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(c) = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$$

$$f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$$

$$\lim_{h\to 0} \frac{\tan(x+h)-\tan x}{h}$$

$\sec^2 x$

$$\lim_{h\to 0} \frac{\ln(e+h)-1}{h}$$

$f(x) = \ln x \quad c = e$

$f'(x) = \frac{1}{x}$

$f'(e) = 1/e$

$$\lim_{x\to 8} \frac{\sqrt[3]{x}-2}{x-8}$$

$f(x) = \sqrt[3]{x} \quad c = 8$

$$\frac{d}{dx}\left(2xy - y^2 + x^3 = 0\right)$$

$$2x\frac{dy}{dx} + y \cdot 2 - 2y\frac{dy}{dx} + 3x^2 = 0$$

$$\frac{dy}{dx} = \frac{-2y - 3x^2}{2x - 2y}$$

A sphere's radius is increasing at a rate of 2 cm/min. At what rate is the volume increasing when the radius is 6 cm?

$\frac{dr}{dt} = 2 \text{ cm/min}$

$\left.\frac{dV}{dt}\right|_{r=6\text{cm}} = \_\_\_\_\_\_$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi(6)^2(2)$

$= 288\pi \text{ cm}^3/\text{min}$

$$f(x) = x^3 + x$$

$$10 = x^3 + x$$

$$2 = x$$

$$f'(x) = 3x^2 + 1$$

$$f'(2) = 13$$

$$(f^{-1})'(10) = \frac{1}{13}$$

$$f^{-1} : (10, \quad)$$

$$f : (\quad , 10)$$

$$y = \arcsin x^2$$

$$y' = \frac{2x}{\sqrt{1-x^4}}$$

$$\frac{u'}{\sqrt{1-u^2}}$$