Rates of change and Rectilinear Motion

- Application of the Derivative: rates of change, velocity
slopes.

- Rate of Change(definition): change in y ÷ change in x
$\Delta y / \Delta x$, (slope)

Examples:

- slope of tangent line
- $f'(2)$
- rate at $t=2$

- Rectilinear Motion Problems – When we talk about these types of problems, we often talk about three types of functions:

1. position:
   Notation: $s(t)$, $f(t)$
   Application(s): finding a position at a given time

   meters

2. Velocity:
   Notation: $v(t)$, $s'(t)$, $f'(t)$

   meters/sec

   Application(s): is the particle moving left or right?
   find the velocity at $t=2$.

3. acceleration:
   Notation: $a(t)$, $v'(t)$, $s''(t)$, $f''(t)$

   meters/sec^2

   Application(s): find the acceleration at $t=2$

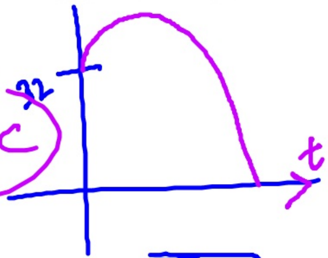
Ex 3: At time t=0 seconds a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$.

a) When does the diver hit the water?

$h=0$

$$0 = -16t^2 + 16t + 32$$
$$0 = -16(t^2 - t - 2)$$
$$0 = -16(t-2)\cancel{(t+1)}$$

$t = 2\text{sec}$



b) What is the diver's velocity at impact?

$$s'(t) = -32t + 16$$
$$s'(2) = -32(2) + 16$$
$$= -64 + 16$$
$$= -48 \text{ ft/sec}$$

P
V
A
J

- Average Velocity vs. Instantaneous Velocity
  - Average Velocity: slope on an interval (a,b)

    need 2 points

    Formula: $\frac{s(b)-s(a)}{b-a}$
  - Instantaneous Velocity: slope at a specific point

    Formula: $s'(t)$ or $v(t)$

- Speed: $|v(t)|$

- Rest: $v(t) = 0$

- Left and Right Motion:
  - Left: $v(t) < 0$ (negative)
  - Right: $v(t) > 0$ (positive)
  - Changes Direction: $v(t)$ changes signs.

Ex 4: A billiard ball is dropped from a height of 100 ft, its height *s* at time *t* is given by the position function $s(t) = -16t^2 + 100$, where *s* is measured in feet and *t* is measured in seconds.

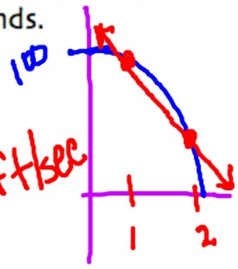
a) Find the average velocity over time interval [1, 2].

$s(1) = 84$

$s(2) = 36$

$(1, 84)\ (2, 36)$

$$\frac{s(2)-s(1)}{2-1} = \frac{36-84}{2-1} = -48 \text{ ft/sec}$$



b) Find the Instantaneous velocity at the endpoints of the interval.

$s'(1) = \underline{-32 \text{ ft}}/\text{sec}$ $\quad s'(2) = \underline{-64 \text{ ft}}/\text{sec}$

$s'(t) = -32t$

c) Find the speed at the endpoints of the interval.

$|s'(1)| = 32 \text{ ft/sec}$

$|s'(2)| = 64 \text{ ft/sec}$

sec meters

Ex 5: A particle starts at time t=0 and moves along the x-axis so that its position at any time $t \ge 0$ is given by $x(t) = (t-1)^3(2t-3)$.

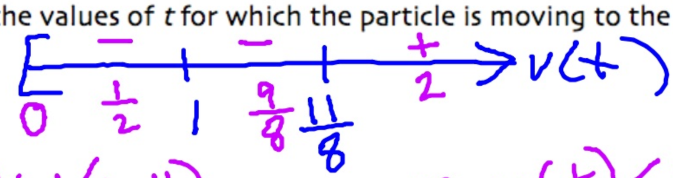
a) Find the velocity of the particle at any time $t \ge 0$. Simplify.

$x'(t) = (t-1)^2(8t-11)$

b) Determine the values of t for which the particle is at rest.

$0 = (t-1)^2(8t-11)$ $\quad t = 1, \frac{11}{8}$ sec

c) Determine the values of t for which the particle is moving to the left. JYA.



$(0,1) \cup (1, \frac{11}{8})$ because $v(t) < 0$ on these intervals d.) $\left(\frac{11}{8}, \infty\right)$ because $v(t) > 0$ on this interval.

e.) $t = \frac{11}{8}$ because $v(t)$ changes signs at this time.

$$x(t) = (t-1)^3(2t-3)$$

$$x'(t) = \underline{(t-1)^3 \cdot 2} + \underline{(2t-3) \cdot 3(t-1)^2 \cdot 1}$$

$$= (t-1)^2\left(2(t-1) + 3(2t-3)\right)$$

$$x'(t) = (t-1)^2(8t-11)$$

On the HW, assume time ≥ 0