

5.7: Last type of Integration

$$\textcircled{1} \int \frac{dx}{x^2 - 6x + 18} = \int \frac{dx}{\square^2 + \square^2}$$

Complete the square

$$\begin{aligned} (x^2 - 6x + 9) + 18 - 9 \\ (x-3)^2 + 9 \end{aligned}$$

$$\int \frac{dx}{(x-3)^2 + 9} \quad \begin{array}{l} u = x-3 \\ du = dx \\ a = 3 \end{array}$$

$$\frac{1}{3} \arctan \frac{x-3}{3} + C$$

$$\textcircled{2} \int \frac{dx}{x^2 - 10x + 30} = \int \frac{dx}{(x-5)^2 + 5}$$

$$\frac{(x^2 - 10x + 25) + 30 - 25}{(x-5)^2 + 5}$$

$$\begin{aligned} u &= x-5 \\ du &= dx \\ a &= \sqrt{5} \end{aligned}$$

$$\frac{1}{\sqrt{5}} \arctan \frac{x-5}{\sqrt{5}} + C$$

$$\begin{aligned}
 \textcircled{3} \int \frac{dx}{\sqrt{2x-x^2}} &= \int \frac{dx}{\sqrt{1-(x-1)^2}} \\
 2x-x^2 & \\
 -x^2+2x & \\
 -(x^2-2x+1)+1 & \\
 -(x-1)^2+1 &
 \end{aligned}$$

arcsin(x-1) + C

$$\textcircled{4} \int \frac{dx}{\sqrt{-x^2 - 12x}}$$

$$-x^2 - 12x$$

$$-(x^2 + 12x + 36) + 36$$

$$-(x+6)^2 + 36$$

$$= \int \frac{dx}{\sqrt{36 - (x+6)^2}}$$

$$\arcsin \frac{x+6}{6} + C$$

$$\textcircled{5} \int \frac{X+1}{X^2+16} dx = \int \frac{X}{X^2+16} dx + \int \frac{1}{X^2+16} dx$$
$$\frac{1}{2} \ln|X^2+16| + \frac{1}{4} \arctan \frac{X}{4} + C$$

$$\textcircled{6} \int \frac{x+4}{\sqrt{9-x^2}} dx = \int \frac{x}{\sqrt{9-x^2}} dx + \int \frac{4}{\sqrt{9-x^2}} dx$$

$$u = 9-x^2$$

$$du = -2x dx$$

$$4 \int \frac{1}{\sqrt{9-x^2}} dx$$

$$-\frac{1}{2} \int u^{-1/2} du$$

$$-\frac{1}{2} \cdot \frac{u^{1/2}}{1/2}$$

HW Pg. 387:

21, 22, 39, 43

AND review 5.6 and 5.7

$$-\sqrt{9-x^2} + 4 \arcsin \frac{x}{3} + C$$