L'Hopital's WKST

Directions: For each limit identify which indeterminate form is produced by the limit, if any. Then find the limit appropriately. For any limit that does not exist, explain why.

1. $\lim\limits_{x\to 0}\dfrac{x}{\tan x}$

2. $\lim\limits_{x\to 1}\dfrac{1-\dfrac{1}{x}}{1-\dfrac{1}{x^2}}$

3. $\lim\limits_{x\to\infty}\dfrac{\log_2 x}{\log_3 x}$

4. $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{\tan x}$

5. $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{3x}\left(1+\dfrac{1}{x}\right)^{-x}$

6. $\lim\limits_{h\to 0}\dfrac{8\left(\dfrac{1}{2}+h\right)^8-8\left(\dfrac{1}{2}\right)^8}{h}$

7. $\lim\limits_{m\to 0}\dfrac{1}{m}\ln\left(\dfrac{2+m}{2}\right)$

8. $\lim\limits_{x\to\infty}\dfrac{4x^2}{10{,}000x+x^2}$

9. $\lim\limits_{x\to 0}(1+2x)^{\csc x}$

10. $\lim\limits_{x\to\pi^+}\dfrac{1}{\sin(x-\pi)}$

11. $\lim\limits_{x\to\pi^+}\dfrac{2x-2\pi}{\sin(x-\pi)}$

12. $\lim\limits_{x\to\pi^+}\sin\left(\dfrac{1}{x-\pi}\right)$

13. $\lim\limits_{x\to\infty}\cos x$

14. $\lim\limits_{x\to\infty}\dfrac{\cos x}{x}$

15. $\lim\limits_{x\to 0}x\cot(2x)$

16. $\lim\limits_{x\to\infty}x\ln\left(\dfrac{x+1}{x-1}\right)$

17. $\lim\limits_{x\to 0^-}x^3e^{1/x}$

18. $\lim\limits_{x\to 0^+}x^3e^{1/x}$

19. $\lim\limits_{x\to\infty}\dfrac{x^2}{e^{-x}}$

20. $\lim\limits_{x\to 0}\dfrac{x}{x}$

21. $\lim\limits_{x\to 4}\dfrac{x^2-x-12}{x^2-11x+28}$

22. $\lim\limits_{x\to 7}\dfrac{x^2-x-12}{(x-4)(x-7)^2}$

23. $\lim\limits_{x\to 0^+}\left(\dfrac{10}{x}-\dfrac{3}{x^2}\right)$

24. $\lim\limits_{x\to\pi}\dfrac{x-\pi}{\tan x}$

25. $\lim\limits_{x\to\pi}\dfrac{x}{\tan x}$

26. $\lim\limits_{x\to\infty}x\sin\dfrac{1}{x}$

27. $\lim\limits_{x\to 0^+}x^{1/x}$

28. $\lim\limits_{x\to\infty}x^{1/x}$

29. $\lim\limits_{x\to 2^+}\left(\dfrac{8}{x^2-4}-\dfrac{x}{x-2}\right)$

30. $\lim\limits_{x\to 1^+}\dfrac{\int_1^x\cos\theta\,d\theta}{x-1}$

ANSWERS

1. $\frac{0}{0}$, 1
2. $\frac{0}{0}$, $\frac{1}{2}$
3. $\frac{\infty}{\infty}$, $\frac{\ln 3}{\ln 2}$ or $\log_2 3$
4. $\frac{0}{0}$, 2
5. 1^{∞}, e^2
6. $\frac{0}{0}$, $\frac{1}{2}$
7. $\infty \cdot 0$, $\frac{1}{2}$
8. $\frac{\infty}{\infty}$, 4
9. 1^{∞}, e^2
10. none, ∞
11. $\frac{0}{0}$, 2
12. none, The limit DNE because the graph of $y = \sin\left(\frac{1}{x-\pi}\right)$ oscillates between values $[-1,1]$ at $x = \pi$.
13. none, The limit DNE because the graph of $y = \cos x$ oscillates between values $[-1,1]$ at $x \to \infty$.
14. None, 0
15. $0 \cdot \infty$, $\frac{1}{2}$
16. $\infty \cdot 0$, 2
17. none, 0
18. $0 \cdot \infty$, ∞
19. none, ∞
20. $\frac{0}{0}$, 1
21. $\frac{0}{0}$, $-\frac{7}{3}$
22. none, ∞
23. $\infty - \infty$, $-\infty$
24. $\frac{0}{0}$, 1
25. none, The limit DNE because $\lim_{x \to \pi^-} \frac{x}{\tan x} = -\infty \neq \lim_{x \to \pi^+} \frac{x}{\tan x} = \infty$.
26. $\infty \cdot 0$, 1
27. none, 0
28. ∞^0, 1
29. $\infty - \infty$, $-\frac{3}{2}$
30. $\frac{0}{0}$, $\cos 1$