The Logistic Equation

-What is the Logistic Equation?_____

_____

- The Differential Equation:

$$\frac{dy}{dt} = \_\_\_\_\_$$

or

$$\frac{dy}{dt} = \_\_\_\_\_$$

where:

y: _____

k: _____

L: _____

- The Solution: $f(t) = \_\_\_\_\_$

where:

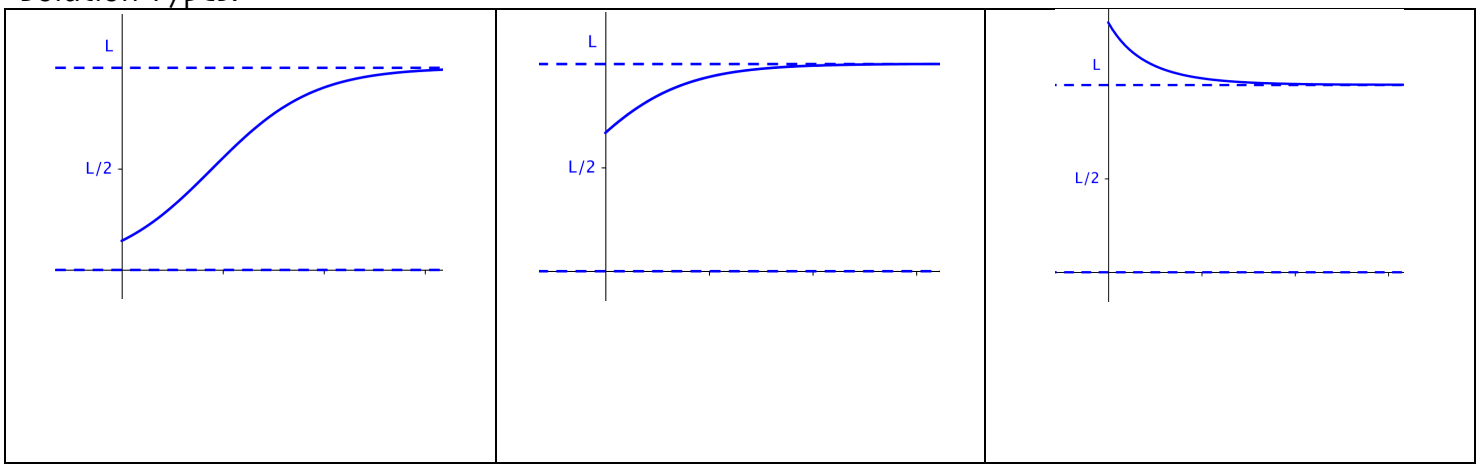
y: _____

b: _____

L: _____

c: _____

Solution Types:



$$\lim_{t \to \infty} f(t) = \_\_\_\_\_$$

$$\lim_{t \to \infty} f'(t) = \_\_\_\_\_$$

Ex 1: A state game commission releases 40 elk into a game refuge. After 5 years the elk population is 104. The commission believes that the environment can support no more than 4000 elk. The growth rate of the elk population, *p*, is

$$\frac{dP}{dt} = kp\left(1 - \frac{p}{4000}\right),$$

a) Write a model for the elk population in terms of *t*.
b) Use the model to estimate the elk population after 15 years. Round to the nearest elk.
c) Find the limit of the model as $t \to \infty$.

Ex 2: The growth rate of a population, *p*, of deer in a newly established wild-life is modeled by the differential equation

$$\frac{dP}{dt} = 0.008P(100 - P)$$

a) What is the carrying capacity for deer?
b) What is the deer population when the population is growing the fastest?
c) What is the rate of change of the population when it is growing the fastest?

Ex 3:

Let f be a function with $f(4) = 1$ such that all points (x, y) on the graph of f satisfy the differential equation

$$\frac{dy}{dx} = 2y(3 - x).$$

Let g be a function with $g(4) = 1$ such that all points (x, y) on the graph of g satisfy the logistic differential equation

$$\frac{dy}{dx} = 2y(3 - y).$$

(a) Find $y = f(x)$.

(b) Given that $g(4) = 1$, find $\lim_{x\to\infty} g(x)$ and $\lim_{x\to\infty} g'(x)$. (It is not necessary to solve for $g(x)$ or to show how you arrived at your answers.)

(c) For what value of y does the graph of g have a point of inflection? Find the slope of the graph of g at the point of inflection. (It is not necessary to solve for $g(x)$.)

Ex 4:
Suppose the population of bears in a national park grows according to the logistic differential equation $\frac{dP}{dt} = 5P - 0.002P^2$, where P is the number of bears at time t in years.

a) If P(0)=100, then $\lim_{t\to\infty} P(t) = ?$ For what values of P is the graph of P increasing? Decreasing? Explain.

b) If P(0)=1500, then $\lim_{t\to\infty} P(t) = ?$ For what values of P is the graph of P increasing? Decreasing? Explain.

c) If P(0)=3000, then $\lim_{t\to\infty} P(t) = ?$ For what values of P is the graph of P increasing? Decreasing? Explain.

Ex: 5
A conservation organization releases 25 Florida Panthers into a game refuge. After 2 years, there are 39 panthers in the preserve. The Florida preserve has a carrying capacity of 200 panthers.
a) Write a logistic equation that models the population of panthers in the preserve.
b) Find the population after 5 years.
c) When will the population reach 100?
d) At what time is the panther population growing most rapidly?

Ex 6:

The spread of a disease through a community can be modeled with the logistic equation $y = \dfrac{600}{1+59e^{-0.1t}}$, where y is the number of people infected after t days. How many people are infected when the disease is spreading the fastest?

(A) 10 (B) 59 (C) 60 (D) 300 (E) 600

Ex: 7

The population $P(t)$ of a species satisfies the logistic differential equation $\dfrac{dP}{dt} = P\left(2 - \dfrac{P}{5000}\right)$, where the initial population is $P(0) = 3000$ and t is the time in years. What is $\lim\limits_{t\to\infty} P(t)$?

(A) 2500 (B) 3000 (C) 4200 (D) 5000 (E) 10,000

Ex 8:

Suppose a population of wolves grows according to the logistic differential equation $\dfrac{dP}{dt} = 3P - 0.01P^2$, where P is the number of wolves at time t, in years. Which of the following statements are true?

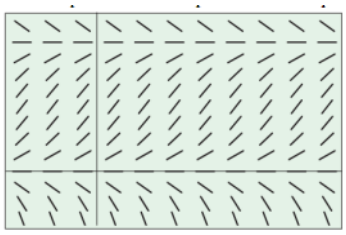
I. $\lim\limits_{t\to\infty} P(t) = 300$

II. The growth rate of the wolf population is greatest when $P = 150$.

III. If $P > 300$, the population of wolves is increasing.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

Ex 9:



[–3, 8] by [–50, 150]

(A) $\dfrac{dy}{dx} = 0.01x(120 - x)$ (B) $\dfrac{dy}{dx} = 0.01y(120 - y)$ (C) $\dfrac{dy}{dx} = 0.01y(100 - x)$

(D) $\dfrac{dy}{dx} = \dfrac{120}{1+60e^{-1.2x}}$ (E) $\dfrac{dy}{dx} = \dfrac{120}{1+60e^{-1.2y}}$

Ex 10:

A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right).$$

(a) If $P(0) = 3$, what is $\lim\limits_{t\to\infty} P(t)$?

If $P(0) = 20$, what is $\lim\limits_{t\to\infty} P(t)$?

(b) If $P(0) = 3$, for what value of P is the population growing the fastest?

(c) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right).$$

Find $Y(t)$ if $Y(0) = 3$.

(d) For the function Y found in part (c), what is $\lim\limits_{t\to\infty} Y(t)$?

ANSWERS

1.

a) $y = \dfrac{4000}{1+99e^{-0.194t}}$

b) 629

c) 4000

2.

a) 100

b) 50

c) 20

3.

a) $y = e^{-x^2+6x-8}$

b) $\lim\limits_{t\to\infty} g(x) = 3$, $\lim\limits_{t\to\infty} g'(x) = 0$

c) $y = \dfrac{3}{2}$, $\left.\dfrac{dy}{dx}\right|_{y=3/2} = \dfrac{9}{2}$

4.

a) $\lim\limits_{t\to\infty} P(t) = 2500$, For t>o the graph of P is increasing since the initial condition is less than the carrying capacity.

b) $\lim\limits_{t\to\infty} P(t) = 2500$, For t>o the graph of P is increasing since the initial condition is less than the carrying capacity.

c) $\lim\limits_{t\to\infty} P(t) = 2500$, For t>o the graph of P is decreasing since the initial condition is more than the carrying capacity.

5.

a) $y = \dfrac{200}{1+7e^{-0.264t}}$

b) 69.695

c) 7.369 years

d) 7.369 years

6. D

7. E

8. C

9. B

10.

a) $\lim\limits_{t\to\infty} P(t) = 12$ (for both limits)

b) 6

c) $y = 3e^{\frac{1}{5}\left(t-\frac{t^2}{24}\right)}$

d) o