AP Calculus - Linear Approximations

I. Use linear approximation at $x = 2$ to estimate the value of $f(a)$ for the given function. Then, state if the approximation is an overestimate or an underestimate and explain.

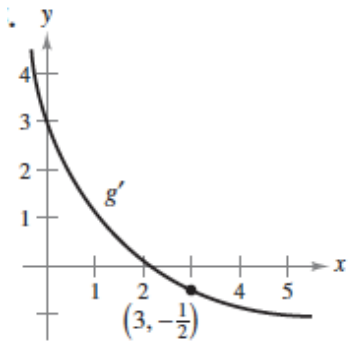
1. $f(x) = \frac{6}{x^2}$; $a = 1.9$
2. $f(x) = x^5$; $a = 2.1$
3. $f(x) = \sqrt{x+7}$; $a = 1.99$

II. Estimate the value of the expression using a linear approximation.

4. $\sqrt{63.9}$
5. $\sqrt[3]{-65}$
6. $(-2.98)^3$

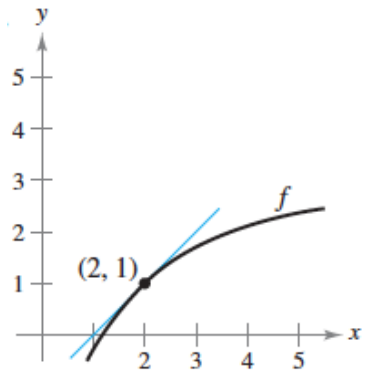
III. Use the graph of g' to approximate g(2.93) and g(3.1) given that g(3) = 8. Is the approximation an underestimate or overestimate? Explain.

7.



IV. Use a linear approximation and the graph of f to approximate f(1.8) and f(2.1). Is the approximation an underestimate or overestimate? Explain.

8.



Answers

1 $1\frac{13}{20}$ = 1.65; underestimate since f″(x)>0 on [1.9, 2]

2 40; underestimate since f″(x)>0 on [2, 2.1]

3 $2\frac{599}{600}$; overestimate since f″(x)<0 on [1.99, 2]

4 $7\frac{159}{160}$

5 $-4\frac{1}{48}$

6 $-26\frac{23}{50}$ = -26.46

7 g(2.93) ≈ $8\frac{7}{200}$ = 8.035; overestimate since g′ is decreasing on [2.93, 3] (that is g is concave down on [2.93, 3])

g(3.1) ≈ $7\frac{19}{20}$ =7.95; overestimate since g′ is decreasing on [3, 3.1] (that is g is concave down on [3, 3.1])

8 f(1.8) ≈ 0.8; overestimate since f is concave down on [1.8, 2]
f(2.1) ≈ 1.1; overestimate since f is concave down [2, 2.1]