1st derivative and Test for Concavity Explanations

$f(x) = 4x^3 - 2x^2$

1st derivative test: to find relative maximums and/or minimums and intervals where f(x) is increasing or decreasing.

$f'(x) = 12x^2 - 4x$ Find the derivative

$0 = 4x(3x-1)$ Find where the derivative is zero or undefined.

```
        +              -              +
<-------------------------------------------------> f'
               0              1/3
               rel.           rel.
               max            min

 increas.          decr.       increas.
<-------------------------------------------------> f
               0              1/3
```

f is increasing on $(-\infty,0)$ and $(1/3,\infty)$ because f' > 0 on these intervals.
f is decreasing on $(0,1/3)$ because f' < 0 on these intervals.
There is a relative maximum at (0, 0) because the sign of f' is changing from positive to negative at this point.
There is a relative minimum at (1/3, -.074) because the sign of f' is changing from negative to positive at this point.

Test for concavity and P.O.I.: to find where f(x) is concave up and/or down and finding the points of inflection

$f''(x) = 24x - 4$ Find the second derivative

$0 = 24x - 4$ Find where the second derivative is zero or undefined.

$0 = 4(6x-1)$

```
          -              +
<-------------------------------------> f''
                1/6

          CCD            CCU
<-------------------------------------> f
                1/6
```

The function is concave down on the interval $(-\infty,1/6)$ because f'' < 0 on this interval.
The function is concave up on the interval $(1/6,\infty)$ because f'' > 0 on this interval.

There is a point of inflection at (1/6, -.037) because f'' changes signs at this point.