Inverse Functions – Worksheet Part 1

I.

1. How can you show two functions are inverses algebraically?
2. How can you show two functions are inverses graphically?
3. How can you show two functions are inverses numerically?

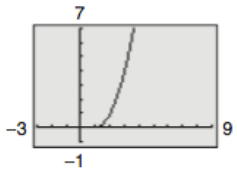
II.

a) Show that f & g are inverse functions algebraically. Justify your answer.
b) Show that f & g are inverse functions graphically. Justify your answer.

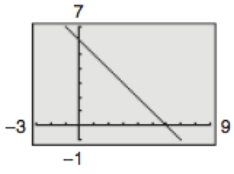
4. (Graph by hand) $f(x) = -\frac{7}{2}x - 3 \qquad g(x) = -\frac{2x+6}{7}$
5. (Graph on the Calculator) $f(x) = x^3 + 5 \qquad g(x) = \sqrt[3]{x-5}$

III. Match the graph of the function with the graph of its inverse function.

(a)



(b)



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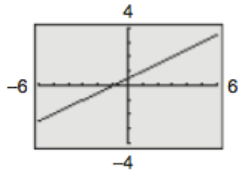

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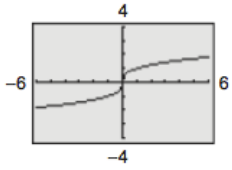

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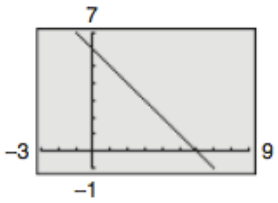

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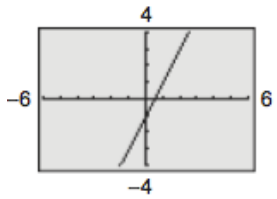

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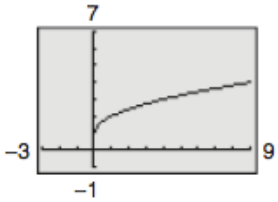

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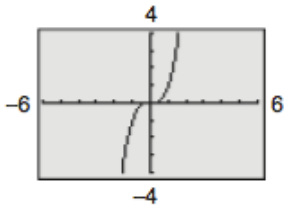


IV. Graph each function. Then use the graph to determine whether the inverse is a function. Explain your reasoning.

10. $f(x) = 3x + 1$
11. $f(x) = 3(x-1)^2 + 2$
12. $f(x) = x^3$
13. $f(x) = \sqrt{x+3} - 1$
14. $f(x) = x^4 - 8x^2 + 9$

V. Are f & g inverse functions? Prove your answer algebraically. Justify your answer.

15. $f(x) = 4 - \frac{3}{2}x,\ g(x) = \frac{1}{2}x + \frac{3}{2}$
16. $f(x) = -\frac{2}{x+1},\ g(x) = -\frac{2}{x} - 1$
17. $f(x) = \frac{x^4 - 1}{5},\ g(x) = \sqrt[4]{5x+1}$
18. $f(x) = \sqrt{x+4},\ g(x) = x^2 - 4$

VI. Could f & g be inverse functions? Prove your answer numerically. Justify your answer.

19.

x	$f(x)$
2	-4
0	5
1	10

x	$g(x)$
-4	2
5	0
10	1

Inverse Functions – Worksheet Part 2

VII. Find the inverse function, if it exists.

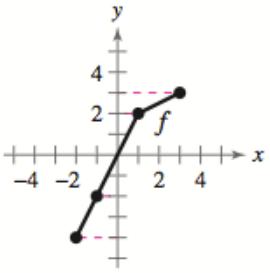
1. $f(x)=5x+12$
2. $h(x)=-\frac{3x^3}{2}-4$
3. $f(x)=\frac{7x+18}{2}$
4. $g(x)=2(x-2)^3$
5. $g(x)=\frac{5}{x}-3$
6. $h(x)=\frac{x+1}{x-1}$

VIII.

7.

a. Use the graph of the function to complete the table for f^{-1}.

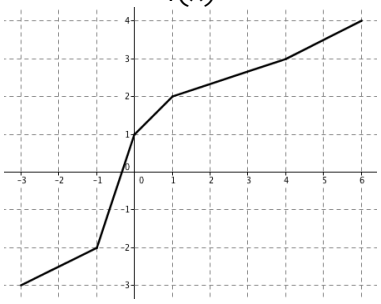
b. Then use the table to sketch f^{-1}.



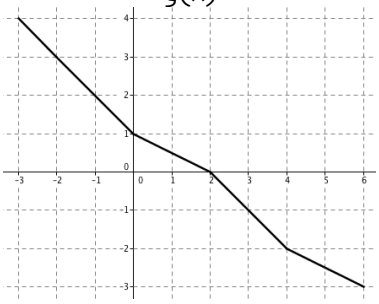
x	$f^{-1}(x)$
-4	
-2	
2	
3	

IX. Use the graphs of *f* and *g* to evaluate each expression

f(x)



g(x)

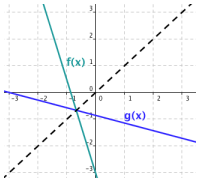


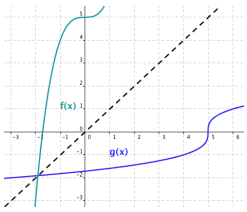
8. $f^{-1}(1)$
9. $(g^{-1})(0)$
10. $(f\circ g)(0)$
11. $g(f(4))$
12. $(f^{-1}\circ g)(0)$
13. $(g^{-1}\circ f)(-1)$
14. $(f\circ g^{-1})(2)$
15. $(f^{-1}\circ g^{-1})(-2)$

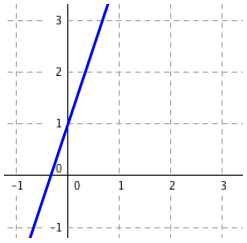
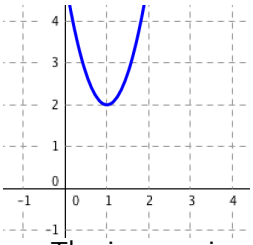
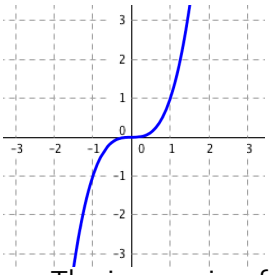
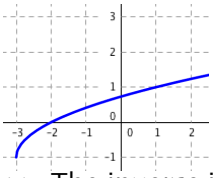
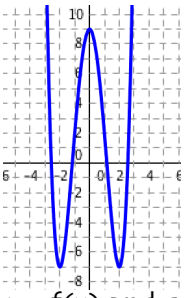
X. Use the functions $f(x)=\frac{1}{8}x-3,\ g(x)=x^3,\ h(x)=x^3+x$ to find the indicated values.

16. $(f^{-1})(1)$
17. $g^{-1}(0)$
18. $(f^{-1}\circ g^{-1})(0)$
19. $(f^{-1}\circ f^{-1})(6)$
20. $h^{-1}(10)$
21. Challenge: $(f\circ g)^{-1}(x)$

ANSWERS: PART 1

1. Show $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.
2. Two functions are inverses if their graphs are reflections about the line y=x.
3. If f(x) contains points (x, y) and g(x) contains points (y, x), then f(x) and g(x) are inverses.
4.
   a. Since $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, f & g are inverse functions.
   b. 

5.
   a. Since $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, f & g are inverse functions.
   b. 

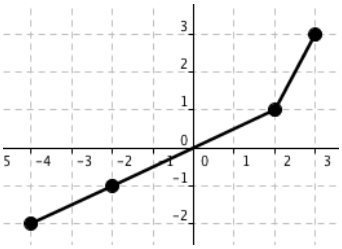
6. B
7. C
8. A
9. D
10. The inverse is a function because f(x) passes the HLT.

11. The inverse is NOT a function because f(x) fails the HLT.

12. The inverse is a function because f(x) passes the HLT.

13. The inverse is a function because f(x) passes the HLT.

14. The inverse is NOT a function because f(x) fails the HLT.

15. f(x) and g(x) are not inverse functions because $f(g(x)) \neq x$ and $g(f(x)) \neq x$.
16. f(x) and g(x) are inverse functions because $f(g(x)) = x$ and $g(f(x)) = x$.
17. f(x) and g(x) are NOT inverse functions; although $f(g(x)) = x$, $g(f(x)) \neq x$.
18. f(x) and g(x) are NOT inverse functions; although $g(f(x)) = x$, $f(g(x)) \neq x$.
19. Yes, f(x) contains points (x, y) and g(x) contains points (y, x).

ANSWERS: PART 2

1. $f^{-1}(x)=\dfrac{x-12}{5}$
2. $h^{-1}(x)=\sqrt[3]{-\dfrac{2x+8}{3}}$
3. $f^{-1}(x)=\dfrac{2x-18}{7}$
4. $g^{-1}(x)=2+\sqrt[3]{\dfrac{x}{2}}$
5. $g^{-1}(x)=\dfrac{5}{x+3}$
6. $h^{-1}(x)=\dfrac{x+1}{x-1}$
7.
   a.

x	$f^{-1}(x)$
-4	-2
-2	-1
2	1
3	3

   b.



8. 0
9. 2
10. 2
11. -1
12. 0
13. 4
14. -2
15. 6
16. 32
17. 0
18. 24
19. 600
20. 2
21. $(f\circ g)^{-1}(x)=2\sqrt[3]{x+3}$