HW Answers #5b

1)

(a) Area $= \int_0^4 \sqrt{x}\,dx + \frac{1}{2}\cdot 2\cdot 2 = \frac{2}{3}x^{3/2}\Big|_{x=0}^{x=4} + 2 = \frac{22}{3}$

$3: \begin{cases} 1: \text{integral} \\ 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$

(b) $y = \sqrt{x} \Rightarrow x = y^2$

$y = 6 - x \Rightarrow x = 6 - y$

Width $= (6 - y) - y^2$

Volume $= \int_0^2 2y\left(6 - y - y^2\right) dy$

$3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

(c) $g'(x) = -1$

Thus a line perpendicular to the graph of g has slope 1.

$f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = 1 \Rightarrow x = \frac{1}{4}$

The point P has coordinates $\left(\frac{1}{4}, \frac{1}{2}\right)$.

$3: \begin{cases} 1: f'(x) \\ 1: \text{equation} \\ 1: \text{answer} \end{cases}$

2)

(a) $r'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$
$r'(t) = 0$ when $t = 1$ and $t = 3$
$r'(t) > 0$ for $0 < t < 1$ and $3 < t < 6$
$r'(t) < 0$ for $1 < t < 3$

Therefore R is moving to the right for $0 < t < 1$ and $3 < t < 6$.

$2 : \begin{cases} 1 : r'(t) \\ 1 : \text{answer} \end{cases}$

(b) $p'(t) = -2 \cdot \frac{\pi}{4} \sin\left(\frac{\pi}{4} t\right)$
$p'(t) = 0$ when $t = 0$ and $t = 4$

$p'(t) < 0$ for $0 < t < 4$
$p'(t) > 0$ for $4 < t < 6$

Therefore the particles travel in opposite directions for $0 < t < 1$ and $3 < t < 4$.

$3 : \begin{cases} 1 : p'(t) \\ 1 : \text{sign analysis for } p'(t) \\ 1 : \text{answer} \end{cases}$

(c) $p''(t) = -2 \cdot \frac{\pi}{4} \cdot \frac{\pi}{4} \cos\left(\frac{\pi}{4} t\right)$
$p''(3) = -2\left(\frac{\pi}{4}\right)^2 \cos\left(\frac{3\pi}{4}\right) = \frac{\pi^2}{8} \cdot \frac{\sqrt{2}}{2} > 0$
$p'(3) < 0$
Therefore particle P is slowing down at time $t = 3$.

$2 : \begin{cases} 1 : p''(3) \\ 1 : \text{answer with reason} \end{cases}$

(d) $\frac{1}{2} \int_1^3 \left| p(t) - r(t) \right| dt$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$