AP Calculus
Answers for FR 2c

1)

(a) Area $= \int_0^2 [g(x) - f(x)]\,dx$

$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right] dx$$

$$= \left[4 \cdot \frac{4}{\pi}\sin\left(\frac{\pi}{4}x\right) - \left(\frac{2x^3}{3} - 3x^2 + 4x\right)\right]_0^2$$

$$= \frac{16}{\pi} - \left(\frac{16}{3} - 12 + 8\right) = \frac{16}{\pi} - \frac{4}{3}$$

$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$

$$= \pi\int_0^2 \left[\left(4 - \left(2x^2 - 6x + 4\right)\right)^2 - \left(4 - 4\cos\left(\frac{\pi}{4}x\right)\right)^2\right] dx$$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

(c) Volume $= \int_0^2 [g(x) - f(x)]^2\,dx$

$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right]^2 dx$$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

2)

(a) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(1,\,0)} = e^0\left(3\cdot 1^2 - 6\cdot 1\right) = -3$

An equation for the tangent line is $y = -3(x-1)$.

$f(1.2) \approx -3(1.2-1) = -0.6$

$$3:\begin{cases} 1: \frac{dy}{dx} \text{ at the point } (x,\,y) = (1,\,0) \\ 1: \text{tangent line equation} \\ 1: \text{approximation} \end{cases}$$

(b) $\frac{dy}{e^y} = \left(3x^2 - 6x\right)dx$

$\int \frac{dy}{e^y} = \int \left(3x^2 - 6x\right)dx$

$-e^{-y} = x^3 - 3x^2 + C$

$-e^{-0} = 1^3 - 3\cdot 1^2 + C \Rightarrow C = 1$

$-e^{-y} = x^3 - 3x^2 + 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$-y = \ln\left(-x^3 + 3x^2 - 1\right)$

$y = -\ln\left(-x^3 + 3x^2 - 1\right)$

Note: This solution is valid on an interval containing $x = 1$ for which $-x^3 + 3x^2 - 1 > 0$.

$$6:\begin{cases} 1: \text{separation of variables} \\ 2: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } y \end{cases}$$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables