Free Response #1 Answers
Check each question. Add up the points earned and write your score.
Example: 16 out of 36

1)

(a) $v(5.5) = -0.45337,\ a(5.5) = -1.35851$ The speed is increasing at time $t = 5.5$, because velocity and acceleration have the same sign.	2 : conclusion with reason
(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\,dt = 1.949$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) Distance $= \int_0^6 \lvert v(t) \rvert\,dt = 12.573$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$. $v(t)$ changes sign from positive to negative at time $t = b$. $x(b) = 2 + \int_0^b v(t)\,dt = 14.134$ or 14.135	$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2)

(a) $H'(3.5) \approx \dfrac{H(5)-H(2)}{5-2}$ $= \dfrac{52-60}{3} = -2.666$ or -2.667 degrees Celsius per minute	1 : answer
(b) $\dfrac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes. $\dfrac{1}{10}\int_0^{10} H(t)\,dt \approx \dfrac{1}{10}\left(2\cdot\dfrac{66+60}{2} + 3\cdot\dfrac{60+52}{2} + 4\cdot\dfrac{52+44}{2} + 1\cdot\dfrac{44+43}{2}\right)$ $= 52.95$	3 : { 1 : meaning of expression 1 : trapezoidal sum 1 : estimate
(c) $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = -23$ The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.	2 : { 1 : value of integral 1 : meaning of expression
(d) $B(10) = 100 + \int_0^{10} B'(t)\,dt = 34.18275;\quad H(10) - B(10) = 8.817$ The biscuits are 8.817 degrees Celsius cooler than the tea.	3 : { 1 : integrand 1 : uses $B(0) = 100$ 1 : answer

3)

(a) $\lim_{x\to 0^-}(1-2\sin x) = 1$

$\lim_{x\to 0^+} e^{-4x} = 1$

$f(0) = 1$

So, $\lim_{x\to 0} f(x) = f(0)$.

Therefore f is continuous at $x = 0$.

2 : analysis

(b) $f'(x) = \begin{cases} -2\cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$

$-2\cos x \neq -3$ for all values of $x < 0$.

$-4e^{-4x} = -3$ when $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right) > 0$.

Therefore $f'(x) = -3$ for $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right)$.

$3 : \begin{cases} 2 : f'(x) \\ 1 : \text{value of } x \end{cases}$

(c) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$

$= \int_{-1}^{0}(1-2\sin x)\,dx + \int_{0}^{1} e^{-4x}\,dx$

$= \Big[x + 2\cos x\Big]_{x=-1}^{x=0} + \left[-\frac{1}{4}e^{-4x}\right]_{x=0}^{x=1}$

$= (3 - 2\cos(-1)) + \left(-\frac{1}{4}e^{-4} + \frac{1}{4}\right)$

Average value $= \frac{1}{2}\int_{-1}^{1} f(x)\,dx$

$= \frac{13}{8} - \cos(-1) - \frac{1}{8}e^{-4}$

$4 : \begin{cases} 1 : \int_{-1}^{0}(1-2\sin x)\,dx \text{ and } \int_{0}^{1} e^{-4x}\,dx \\ 2 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$

4)

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = -6 - \frac{9\pi}{4}$

$g'(x) = 2 + f(x)$

$g'(-3) = 2 + f(-3) = 2$

$3 : \begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$.

$g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$.

Therefore g has an absolute maximum at $x = \frac{5}{2}$.

$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.

1 : answer with reason

(d) The average rate of change of f on the interval $-4 \le x \le 3$ is $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$.

To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

$2 : \begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$