

Existence Theorems: AP Style Questions

1.

If f is continuous for $a \leq x \leq b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \frac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \leq x \leq b$.

(D) f has a maximum value on $a \leq x \leq b$.

2.

If f is a continuous function on $[a, b]$, which of the following is necessarily true?

(A) f' exists on (a, b) .

(B) If $f(x_0)$ is a maximum of f , then $f'(x_0) = 0$.

(C) $\lim_{x \rightarrow x_0} f(x) = f\left(\lim_{x \rightarrow x_0} x\right)$ for $x_0 \in (a, b)$

(D) $f'(x) = 0$ for some $x \in [a, b]$

(E) The graph of f' is a straight line.

3.

Let g be a continuous function on the closed interval $[0, 1]$. Let $g(0) = 1$ and $g(1) = 0$. Which of the following is NOT necessarily true?

(A) There exists a number h in $[0, 1]$ such that $g(h) \geq g(x)$ for all x in $[0, 1]$.

(B) For all a and b in $[0, 1]$, if $a = b$, then $g(a) = g(b)$.

(C) There exists a number h in $[0, 1]$ such that $g(h) = \frac{1}{2}$.

(D) There exists a number h in $[0, 1]$ such that $g(h) = \frac{3}{2}$.

(E) For all h in the open interval $(0, 1)$, $\lim_{x \rightarrow h} g(x) = g(h)$.

4.

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0, 2]$ if $k =$

- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

5.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x . The function h is given by $h(x) = f(g(x)) - 6$.

- (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.
- (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.