Existence Theorems: MC Style

1.

If f is continuous for $a \leq x \leq b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \leq x \leq b$.

(D) f has a maximum value on $a \leq x \leq b$.

2.

If f is a continuous function on $[a,b]$, which of the following is necessarily true?

(A) f' exists on (a,b).

(B) If $f(x_0)$ is a maximum of f, then $f'(x_0) = 0$.

(C) $\lim_{x \to x_0} f(x) = f\left(\lim_{x \to x_0} x\right)$ for $x_0 \in (a,b)$

(D) $f'(x) = 0$ for some $x \in [a,b]$

(E) The graph of f' is a straight line.

3.

Let g be a continuous function on the closed interval $[0,1]$. Let $g(0) = 1$ and $g(1) = 0$. Which of the following is NOT necessarily true?

(A) There exists a number h in $[0,1]$ such that $g(h) \geq g(x)$ for all x in $[0,1]$.

(B) For all a and b in $[0,1]$, if $a = b$, then $g(a) = g(b)$.

(C) There exists a number h in $[0,1]$ such that $g(h) = \frac{1}{2}$.

(D) There exists a number h in $[0,1]$ such that $g(h) = \frac{3}{2}$.

(E) For all h in the open interval $(0,1)$, $\lim_{x \to h} g(x) = g(h)$.

4.

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3