Ellipses WKST

1. Fill in the blanks.
   a) A(n) _____ is the locus of points, the sum of whose distances from two distinct fixed points is constant.

   b) The chord joining the vertices of an ellipse is called the _____, and its midpoint is the _____ of the ellipse.

   c) The chord perpendicular to the major axis at the center of an ellipse is called the _____ of the ellipse.

   d) The concept of _____ is used to measure the ovalness of an ellipse.

2. Sketch the graph. Identify the center, vertices, co-vertices, foci, major axis length, minor axis length and eccentricity.

   a) $\frac{x^2}{64}+\frac{y^2}{36}=1$

   b) $\frac{x^2}{16}+\frac{y^2}{49}=1$

   c) $\frac{(x+7)^2}{4}+\frac{(y-5)^2}{25}=1$

   d) $\frac{(x-3)^2}{9}+\frac{(y-8)^2}{100}=1$

   e) $9x^2+16y^2=144$

   f) $49(x+2)^2+9(y-1)^2=36$

   g) $3x^2+y^2+6x-8y-11=0$

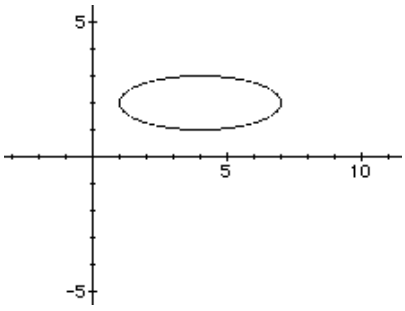
   h) $16x^2+4y^2+96x-8y+84=0$

   i) $4x^2+9y^2-16x+18y-11=0$

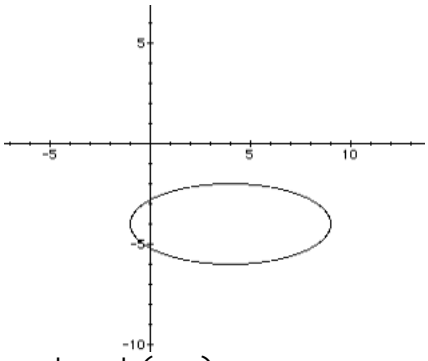
   j) $36x^2+64y^2+108x-128y-431=0$

3. Find the equation of an ellipse in standard form satisfying the given conditions:

   a)



   b)



   c) center (3, –2)
   passes through (–4, –2), (10, –2), (3, 1), and (3, –5)

   d) center at (2, 5)
   longer axis of length 12 and parallel to the *x*–axis
   shorter axis of length 10

   e) Foci $(\pm 3,0)$
   major axis length 12

   f) vertices $(\pm 4,-1)$
   tangent to the *x*-axis

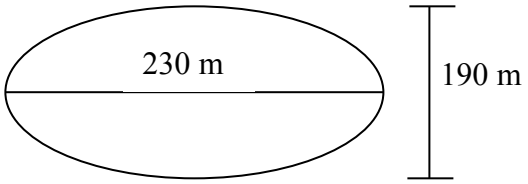
4. Rewrite each equation in general form.

   a) $\frac{x^2}{9}+\frac{(y-1)^2}{25}=1$

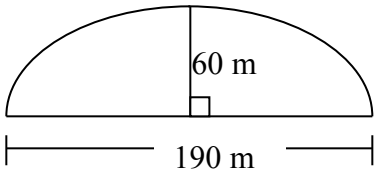
   b) $\frac{(x-3)^2}{64}+\frac{(y+1)^2}{36}=1$

5. B.C. Place Stadium has an air–filled fabric dome roof that forms the shape of an ellipse when viewed from above. Its maximum length is approximately 230 m, its maximum width is approximately 190 m, and its maximum height is approximately 60 m.

   a) Find an equation for the ellipse formed by the base of the roof.

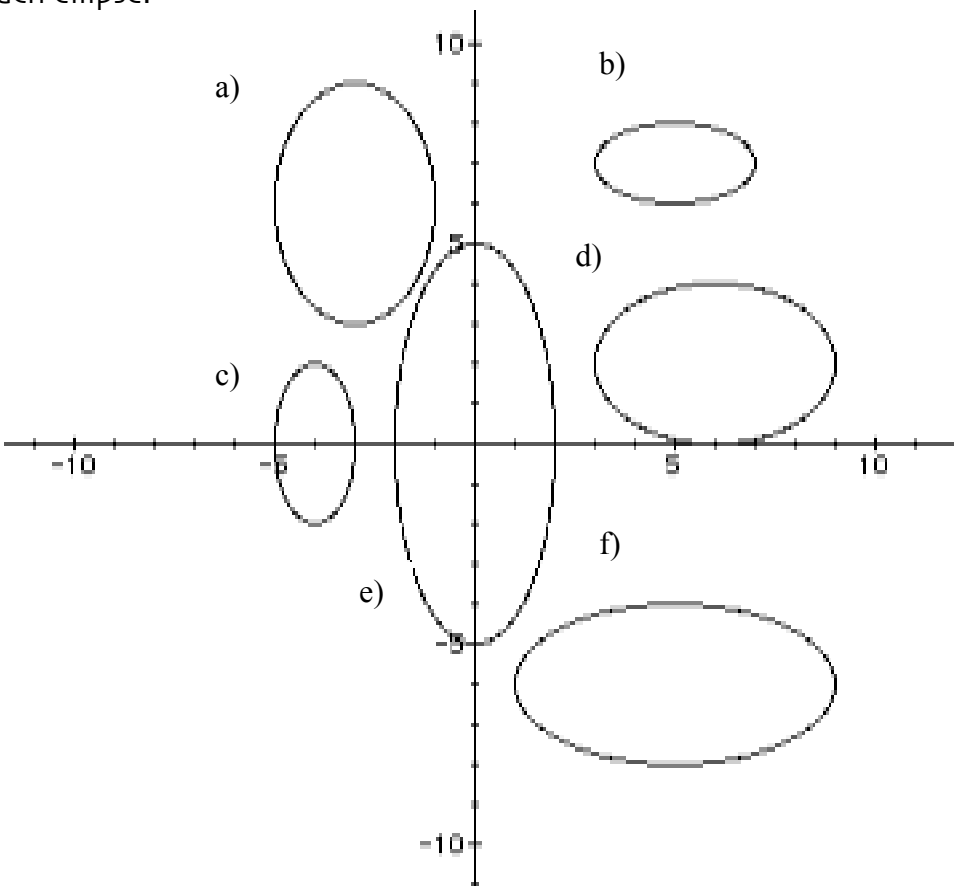


   b) Taking a cross section of the roof at its greatest width results in a semi–ellipse. Find an equation for this semi–ellipse.



   c) The promoters of a concert plan to send fireworks up from a point on the stage that is 30m lower than the center in part b, and 40 m along the major axis of this ellipse from its center. How far is that point on the stage from the roof?

6. Describe the transformations (translations and stretches) that have been applied to the unit circle $x^2 + y^2 = 1$ to produce each of the ellipses below. Then find the standard form of the equation of each ellipse.



ANSWERS

1.

a) ellipse b) major axis, center c) minor axis d) eccentricity

2.

a)

Center: $(0,0)$

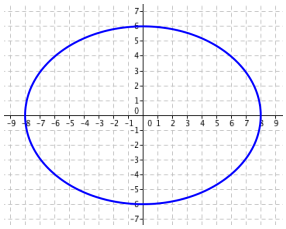
Vertices: $(\pm 8,0)$

Covertices: $(0,\pm 6)$

Foci: $(\pm 2\sqrt{7},0)$

Major: 16
Minor: 12

Ecc: $\frac{2\sqrt{7}}{8}$



b)

Center: $(0,0)$

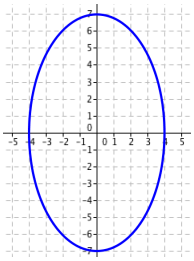
Vertices: $(0,\pm 7)$

Covertices: $(\pm 4,0)$

Foci: $(0,\pm\sqrt{33})$

Major:14
Minor:8

Ecc: $\frac{\sqrt{33}}{7}$



c)

Center: $(-7,5)$

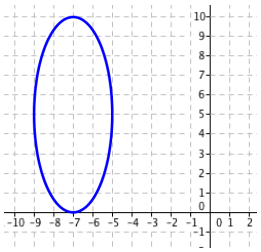
Vertices: $(-7,0),(-7,10)$

Covertices: $(-9,5),(-5,5)$

Foci: $(-7,5\pm\sqrt{21})$

Major: 10
Minor: 4

Ecc: $\frac{\sqrt{21}}{5}$



d)

Center: $(3,8)$

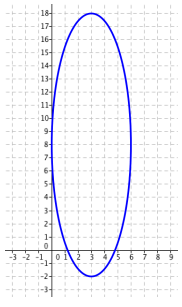
Vertices: $(0,8),(6,8)$

Covertices: $(3,-2),(3,18)$

Foci: $\left(3,8\pm\frac{\sqrt{91}}{10}\right)$

Major: 20
Minor: 6

Ecc: $\frac{\sqrt{91}}{10}$



e)

Center: $(0,0)$

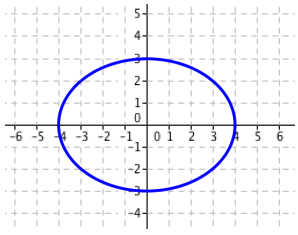
Vertices: $(\pm 4,0)$

Covertices: $(0,\pm 3)$

Foci: $(\pm\sqrt{7},0)$

Major: 8
Minor: 6

Ecc: $\frac{\sqrt{7}}{4}$



f)

Center: $(-2,1)$

Vertices: $\left(-\frac{20}{7},1\right),\left(-\frac{8}{7},1\right)$

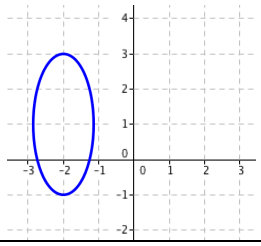
Covertices: $(-2,-1),(-2,3)$

Foci: $\left(-2,1\pm\frac{4\sqrt{10}}{7}\right)$

Major: 4

Minor: $\frac{12}{7}$

Ecc: $\frac{2\sqrt{10}}{7}$



g)

Center: $(-1,4)$

Ver: $\left(-1,4\pm\sqrt{30}\right)$

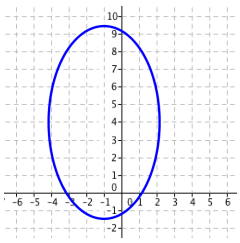
Cover: $\left(-1\pm\sqrt{10},4\right)$

Foci: $\left(-1,4\pm2\sqrt{5}\right)$

Major: $2\sqrt{30}$

Minor: $2\sqrt{10}$

Ecc: $\frac{\sqrt{6}}{3}$



h)

Center: $(-3,1)$

Ver: $(-3,5)$, $(-3,-3)$

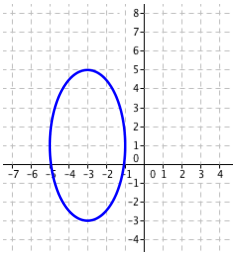
Cover: $(-5,1)$, $(-1,1)$

Foci: $\left(-3,1\pm2\sqrt{3}\right)$

Major: 8

Minor: 4

Ecc: $\frac{\sqrt{3}}{2}$



i)

Center: $(2,-1)$

Ver: $(-1,-1)$, $(5,-1)$

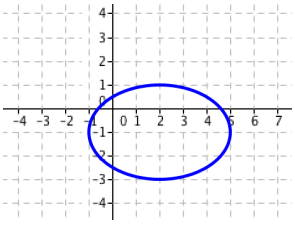
Cover: $(2,1)$, $(2,-3)$

Foci: $\left(2\pm\sqrt{5},-1\right)$

Major: 6

Minor: 4

Ecc: $\frac{\sqrt{5}}{3}$



j)

Center: $\left(-\frac{3}{2},1\right)$

Ver: $\left(-\frac{11}{2},1\right)$, $\left(\frac{5}{2},1\right)$

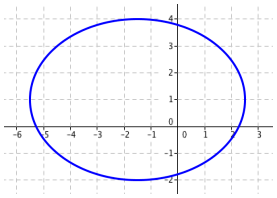
Cover: $\left(-\frac{3}{2},4\right)$, $\left(-\frac{3}{2},-2\right)$

Foci: $\left(-\frac{3}{2}\pm\sqrt{7},1\right)$

Major: 8

Minor:6

Ecc: $\frac{\sqrt{7}}{4}$



3.

a) $\frac{(x-4)^2}{9}+(y-2)^2=1$

b) $\frac{(x-4)^2}{25}+\frac{(y+4)^2}{4}=1$

c) $\frac{(x-3)^2}{49}+\frac{(y+2)^2}{9}=1$

d) $\frac{(x-2)^2}{36}+\frac{(y-5)^2}{25}=1$

e) $\frac{x^2}{36}+\frac{y^2}{27}=1$

f) $\frac{x^2}{16}+(y+1)^2=1$

4.

a) $25x^2+9y^2-18y-216=0$

b) $9x^2+16y^2-54x+32y-479=0$

5.

a) $\frac{x^2}{13225}+\frac{y^2}{9025}=1$

b) $\frac{x^2}{9025}+\frac{y^2}{3600}=1$

c) 84.4 m

6.

a) Vertical stretch by a factor of 3, horizontal stretch by a factor of 2, up 6, and left 3.

$$\frac{(x+3)^2}{4}+\frac{(y-6)^2}{9}=1$$

b) Horizontal stretch by a factor of 2, up 7, and right 5.

$$\frac{(x-5)^2}{4}+(y-7)^2=1$$

c) Vertical stretch by a factor of 2 and left 4.

$$(x+4)^2+\frac{y^2}{4}=1$$

d) Vertical stretch by a factor of 2, horizontal stretch by a factor of 3, up 2, and right 6.

$$\frac{(x-6)^2}{9}+\frac{(y-2)^2}{4}=1$$

e) Vertical stretch by a factor of 5, horizontal stretch by a factor of 2.

$$\frac{x^2}{4}+\frac{y^2}{25}=1$$

f) Vertical stretch by a factor of 4, horizontal stretch by a factor of 2, down 6 and right 5

$$\frac{(x-5)^2}{16}+\frac{(y+6)^2}{4}=1$$