Definite Integrals and Riemann Sums

Definition of a Definite Integral

If f is a continuous function defined on [a,b], and if [a,b] is divided into n equal subintervals of width $\Delta x = \frac{b-a}{n}$, and if $x_k = a + k\Delta x$ is the right endpoint of subinterval k, then the definite integral of f from a to b is:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

Or:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} \left[f\left(a + \frac{(b-a)k}{n}\right)\right]\left(\frac{b-a}{n}\right)$$

Rewrite each definite integral as a limit with sigma notation.

1. $\int_0^3 (6x)\, dx$

2. $\int_0^4 (x^2 + 2)\, dx$

3. $\int_0^2 (4x^2 + 5x)\, dx$

4. $\int_0^5 (-2x^2 - 4x + 3)\, dx$

5. $\int_2^4 7x^3\, dx$

6. $\int_1^4 (x^2 - 3x)\, dx$

7. $\int_{-1}^2 3x^3\, dx$

8. $\int_{-4}^0 (3x + 7)\, dx$

9. $\int_{-2}^{-1} (5x^2 - 6)\, dx$

10. $\int_{-7}^{-2} (x^2 + 4x + 1)\, dx$

Rewrite each as a definite integral.

11. $\lim_{n\to\infty} \sum_{k=1}^{n} \left[3\left(\frac{k}{n}\right)^2 + 2\right]\left(\frac{1}{n}\right)$

12. $\lim_{n\to\infty} \sum_{k=1}^{n} \left[2\left(\frac{k}{n}\right)^3 + 4\left(\frac{k}{n}\right) - 3\right]\left(\frac{1}{n}\right)$

13. $\lim_{n\to\infty} \sum_{k=1}^{n} \left[4\left(2 + \frac{k}{n}\right)^2\right]\left(\frac{1}{n}\right)$

14. $\lim_{n\to\infty} \sum_{k=1}^{n} \left[3\left(2 + \frac{2k}{n}\right)^2 - 7\right]\left(\frac{2}{n}\right)$

15. $\lim_{n\to\infty} \sum_{k=1}^{n} \left[5\left(-2 + \frac{3k}{n}\right)^3 + 2\right]\left(\frac{3}{n}\right)$

16. $\lim_{n\to\infty} \sum_{k=1}^{n} \left[3\left(-5 + \frac{4k}{n}\right)^3 - 2\left(-5 + \frac{4k}{n}\right)^2\right]\left(\frac{4}{n}\right)$

17. $\lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{3}{\left(1+\frac{2k}{n}\right)^2}\right]\left(\frac{2}{n}\right)$

18. $\lim_{n\to\infty} \sum_{k=1}^{n} \left[\sqrt{\left(2 + \frac{k}{n}\right)^2 + 7}\right]\left(\frac{1}{n}\right)$

19. $\lim_{n\to\infty} \sum_{k=1}^{n} \left[1 + \frac{5}{k/n}\right]\left(\frac{1}{n}\right)$

20. $\lim_{n\to\infty} \sum_{k=1}^{n} \left[3\left(\frac{k}{n}\right)\left(7 - \frac{k}{n}\right)^2\right]\left(\frac{1}{n}\right)$

ANSWERS:

1.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[6\left(\frac{3k}{n}\right)\right]\left(\frac{3}{n}\right)$$

2.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[\left(\frac{4k}{n}\right)^2+2\right]\left(\frac{4}{n}\right)$$

3.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[4\left(\frac{2k}{n}\right)^2+5\left(\frac{2k}{n}\right)\right]\left(\frac{2}{n}\right)$$

4.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[-2\left(\frac{5k}{n}\right)^2-4\left(\frac{5k}{n}\right)+3\right]\left(\frac{5}{n}\right)$$

5.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[7\left(2+\frac{2k}{n}\right)^3\right]\left(\frac{2}{n}\right)$$

6.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[\left(1+\frac{3k}{n}\right)^2-3\left(1+\frac{3k}{n}\right)\right]\left(\frac{3}{n}\right)$$

7.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[3\left(-1+\frac{3k}{n}\right)^3\right]\left(\frac{3}{n}\right)$$

8.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[3\left(-4+\frac{4k}{n}\right)+7\right]\left(\frac{4}{n}\right)$$

9.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[5\left(-2+\frac{k}{n}\right)^2-6\right]\left(\frac{1}{n}\right)$$

10.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[\left(-7+\frac{5k}{n}\right)^2+4\left(-7+\frac{5k}{n}\right)+1\right]\left(\frac{5}{n}\right)$$

11. $\int_0^1 (3x^2 + 2)dx$

12. $\int_0^1 (2x^3 + 4x - 3)dx$

13. $\int_0^1 (2x^3 + 4x - 3)dx$

14. $\int_2^4 (2x^2 - 7)dx$

15. $\int_{-2}^1 (5x^3 + 2)dx$

16. $\int_{-5}^{-1} (3x^3 - 2x^2)dx$

17. $\int_1^3 \left(\frac{3}{x^2}\right) dx$

18. $\int_2^3 \left(\sqrt{x^2 + 7}\right)dx$

19. $\int_1^2 \left(1 + \frac{5}{x}\right) dx$

20. $\int_0^1 (3x(7 - x)^2)dx$