

Ch 10/12 Multiple Choice

1.

The area of the region inside the polar curve $r = 4 \sin \theta$ and outside the polar curve $r = 2$ is given by

- (A) $\frac{1}{2} \int_0^{\pi} (4 \sin \theta - 2)^2 d\theta$ (B) $\frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (4 \sin \theta - 2)^2 d\theta$ (C) $\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4 \sin \theta - 2)^2 d\theta$
(D) $\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (16 \sin^2 \theta - 4) d\theta$ (E) $\frac{1}{2} \int_0^{\pi} (16 \sin^2 \theta - 4) d\theta$

2.

The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \leq t \leq 1$, is given by

- (A) $\int_0^1 \sqrt{t^2 + 1} dt$
(B) $\int_0^1 \sqrt{t^2 + t} dt$
(C) $\int_0^1 \sqrt{t^4 + t^2} dt$
(D) $\frac{1}{2} \int_0^1 \sqrt{4 + t^4} dt$
(E) $\frac{1}{6} \int_0^1 t^2 \sqrt{4t^2 + 9} dt$

3.

For time $t > 0$, the position of a particle moving in the xy -plane is given by the parametric equations $x = 4t + t^2$ and $y = \frac{1}{3t+1}$. What is the acceleration vector of the particle at time $t = 1$?

- (A) $\left(2, \frac{1}{32}\right)$
(B) $\left(2, \frac{9}{32}\right)$
(C) $\left(5, \frac{1}{4}\right)$
(D) $\left(6, -\frac{3}{16}\right)$
(E) $\left(6, -\frac{1}{16}\right)$

4.

What is the slope of the line tangent to the polar curve $r = 2\theta$ at the point $\theta = \frac{\pi}{2}$?

- (A) $-\frac{\pi}{2}$ (B) $-\frac{2}{\pi}$ (C) 0 (D) $\frac{\pi}{2}$ (E) 2

5.

Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

- (A) $3\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta$ (B) $3\int_0^{\pi} \cos^2\theta d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta$ (D) $3\int_0^{\frac{\pi}{2}} \cos\theta d\theta$ (E) $3\int_0^{\pi} \cos\theta d\theta$

6.

In the xy -plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \leq t \leq 3$, is a line segment with slope

- (A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

7.

The length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$, for $0 \leq t \leq \frac{\pi}{2}$, is given by

- (A) $\int_0^{\frac{\pi}{2}} \sqrt{3\cos^2 t + 3\sin^2 t} dt$
(B) $\int_0^{\frac{\pi}{2}} \sqrt{-3\cos^2 t \sin t + 3\sin^2 t \cos t} dt$
(C) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t + 9\sin^4 t} dt$
(D) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t} dt$
(E) $\int_0^{\frac{\pi}{2}} \sqrt{\cos^6 t + \sin^6 t} dt$

8.

If $x = e^{2t}$ and $y = \sin(2t)$, then $\frac{dy}{dx} =$

- (A) $4e^{2t}\cos(2t)$ (B) $\frac{e^{2t}}{\cos(2t)}$ (C) $\frac{\sin(2t)}{2e^{2t}}$ (D) $\frac{\cos(2t)}{2e^{2t}}$ (E) $\frac{\cos(2t)}{e^{2t}}$

9.

The asymptotes of the graph of the parametric equations $x = \frac{1}{t}$, $y = \frac{t}{t+1}$ are

- (A) $x = 0, y = 0$ (B) $x = 0$ only (C) $x = -1, y = 0$
 (D) $x = -1$ only (E) $x = 0, y = 1$

10.

The area of the region enclosed by the polar curve $r = 1 - \cos \theta$ is

- (A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π

11.

The area of the region enclosed by the polar curve $r = \sin(2\theta)$ for $0 \leq \theta \leq \frac{\pi}{2}$ is

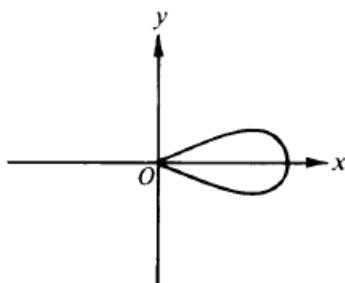
- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{4}$

12.

If $x = t^3 - t$ and $y = \sqrt{3t+1}$, then $\frac{dy}{dx}$ at $t = 1$ is

- (A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{3}{4}$ (D) $\frac{8}{3}$ (E) 8

13.



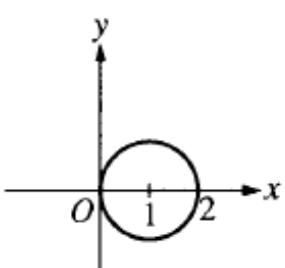
Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve $r = 4 \cos(3\theta)$ shown in the figure above?

- (A) $16 \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos(3\theta) d\theta$ (B) $8 \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos(3\theta) d\theta$ (C) $8 \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos^2(3\theta) d\theta$
 (D) $16 \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos^2(3\theta) d\theta$ (E) $8 \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos^2(3\theta) d\theta$

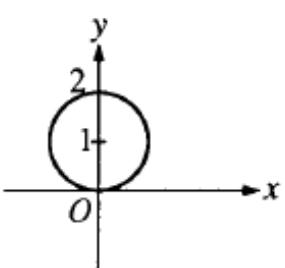
14.

Which of the following represents the graph of the polar curve $r = 2 \sec \theta$?

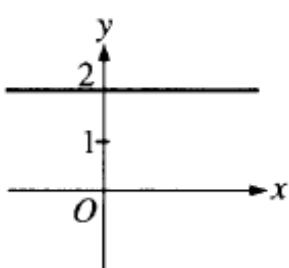
(A)



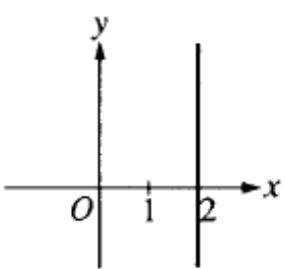
(B)



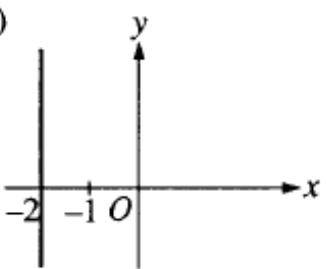
(C)



(D)



(E)



15.

If $x = t^2 + 1$ and $y = t^3$, then $\frac{d^2y}{dx^2} =$

(A) $\frac{3}{4t}$

(B) $\frac{3}{2t}$

(C) $3t$

(D) $6t$

(E) $\frac{3}{2}$

16.

The length of the curve determined by the equations $x = t^2$ and $y = t$ from $t = 0$ to $t = 4$ is

(A) $\int_0^4 \sqrt{4t+1} dt$

(B) $2 \int_0^4 \sqrt{t^2+1} dt$

(C) $\int_0^4 \sqrt{2t^2+1} dt$

(D) $\int_0^4 \sqrt{4t^2+1} dt$

(E) $2\pi \int_0^4 \sqrt{4t^2+1} dt$

17. CALCULATOR

Consider the curve in the xy -plane represented by $x = e^t$ and $y = te^{-t}$ for $t \geq 0$. The slope of the line tangent to the curve at the point where $x = 3$ is

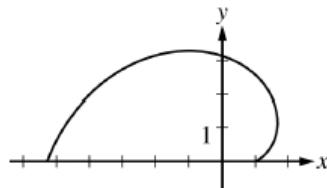
- (A) 20.086 (B) 0.342 (C) -0.005 (D) -0.011 (E) -0.033

18. CALCULATOR

The area of the closed region bounded by the polar graph of $r = \sqrt{3 + \cos \theta}$ is given by the integral

- (A) $\int_0^{2\pi} \sqrt{3 + \cos \theta} d\theta$ (B) $\int_0^\pi \sqrt{3 + \cos \theta} d\theta$ (C) $2 \int_0^{\pi/2} (3 + \cos \theta) d\theta$
(D) $\int_0^\pi (3 + \cos \theta) d\theta$ (E) $2 \int_0^{\pi/2} \sqrt{3 + \cos \theta} d\theta$

19. CALCULATOR



The graph above shows the polar curve $r = 2\theta + \cos \theta$ for $0 \leq \theta \leq \pi$. What is the area of the region bounded by the curve and the x -axis?

- (A) 3.069 (B) 4.935 (C) 9.870 (D) 17.456 (E) 34.912

REVIEW QUESTIONS

20.

The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between $(0, 0)$ and $(4, 2)$. What are the coordinates of this point?

- (A) $(2, 1)$
(B) $(1, 1)$
(C) $(2, \sqrt{2})$
(D) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$
(E) None of the above

21.

$$\int_0^8 \frac{dx}{\sqrt{1+x}} =$$

- (A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

22.

$$\int_0^1 \frac{x^2}{x^2 + 1} dx =$$

- (A) $\frac{4-\pi}{4}$ (B) $\ln 2$ (C) 0 (D) $\frac{1}{2} \ln 2$ (E) $\frac{4+\pi}{4}$

23.

The region bounded by the x -axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and

$x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \leq x \leq k$ is

three times the area of the region for $k \leq x \leq \frac{\pi}{2}$, then $k =$

- (A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

24.

$$\int_0^1 \sqrt{x^2 - 2x + 1} dx$$
 is

- (A) -1
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 1
(E) none of the above

25.

If $\int x^2 \cos x dx = f(x) - \int 2x \sin x dx$, then $f(x) =$

- (A) $2 \sin x + 2x \cos x + C$
(B) $x^2 \sin x + C$
(C) $2x \cos x - x^2 \sin x + C$
(D) $4 \cos x - 2x \sin x + C$
(E) $(2-x^2) \cos x - 4 \sin x + C$

26.

$$\int_1^{\infty} \frac{x}{(1+x^2)^2} dx \text{ is}$$

- (A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) divergent

27.

$$\int \frac{1}{x^2 - 6x + 8} dx =$$

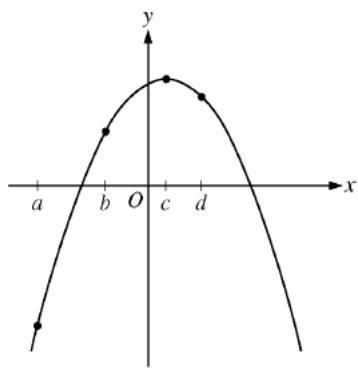
- (A) $\frac{1}{2} \ln \left| \frac{x-4}{x-2} \right| + C$
 (B) $\frac{1}{2} \ln \left| \frac{x-2}{x-4} \right| + C$
 (C) $\frac{1}{2} \ln |(x-2)(x-4)| + C$
 (D) $\frac{1}{2} \ln |(x-4)(x+2)| + C$
 (E) $\ln |(x-2)(x-4)| + C$

28.

If f is a differentiable function such that $f(3) = 8$ and $f'(3) = 5$, which of the following statements could be false?

- (A) $\lim_{x \rightarrow 3} f(x) = 8$
 (B) $\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^-} f(x)$
 (C) $\lim_{x \rightarrow 3} \frac{f(x) - 8}{x - 3} = 5$
 (D) $\lim_{h \rightarrow 0} \frac{f(3+h) - 8}{h} = 5$
 (E) $\lim_{x \rightarrow 3} f'(x) = 5$

29.

Graph of f

The figure above shows the graph of a function f . Which of the following has the greatest value?

- (A) $f(a)$ (B) $f'(a)$ (C) $f'(c)$ (D) $f(d) - f(c)$ (E) $\frac{f(b) - f(a)}{b - a}$

ANSWERS

- | | | | | |
|------|-------|-------|-------|-------|
| 1. D | 7. D | 13. E | 19. D | 25. B |
| 2. C | 8. E | 14. D | 20. B | 26. C |
| 3. B | 9. C | 15. A | 21. E | 27. A |
| 4. B | 10. C | 16. D | 22. A | 28. E |
| 5. A | 11. D | 17. D | 23. C | 29. B |
| 6. A | 12. B | 18. D | 24. C | |