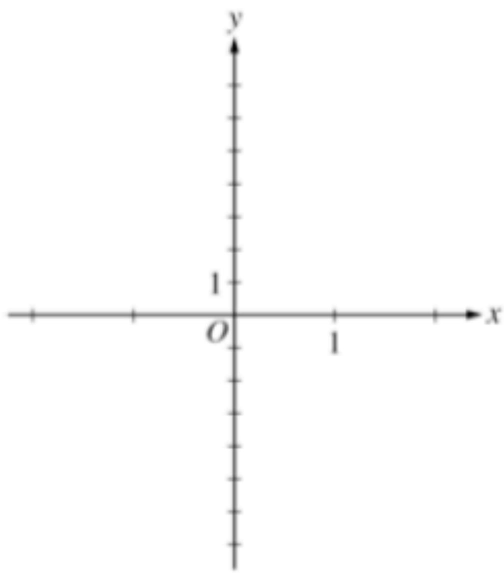
Chapter 9 AP Problems

1. CALCULATOR

A particle moves in the xy-plane so that its position at any time t, for $-\pi \le t \le \pi$, is given by $x(t) = \sin(3t)$ and $y(t) = 2t$.

(a) Sketch the path of the particle in the xy-plane provided. Indicate the direction of motion along the path. **(Note: Use the axes provided in the test booklet.)**



(b) Find the range of $x(t)$ and the range of $y(t)$.

(c) Find the smallest positive value of t for which the x-coordinate of the particle is a local maximum. What is the speed of the particle at this time?

(d) Is the distance traveled by the particle from $t = -\pi$ to $t = \pi$ greater than 5π ? Justify your answer.

2. CALCULATOR

At time t, $0 \le t \le 2\pi$, the position of a particle moving along a path in the xy-plane is given by the parametric equations $x = e^t \sin t$ and $y = e^t \cos t$.

(a) Find the slope of the path of the particle at time $t = \frac{\pi}{2}$.

(b) Find the speed of the particle when $t = 1$.

(c) Find the distance traveled by the particle along the path from $t = 0$ to $t = 1$.

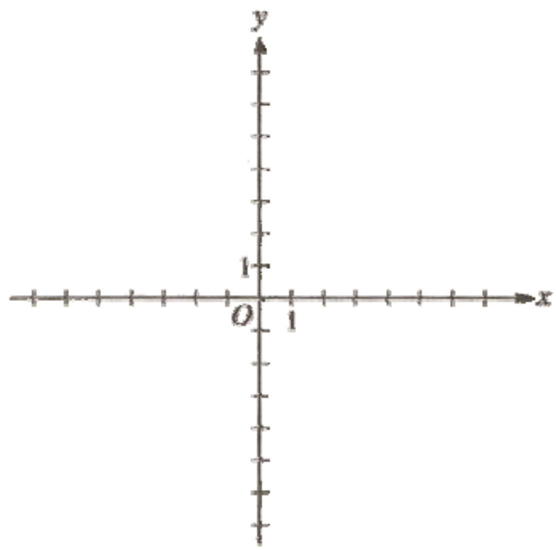
3. CALCULATOR

During the time period from $t = 0$ to $t = 6$ seconds, a particle moves along the path given by $x(t) = 3\cos(\pi t)$ and $y(t) = 5\sin(\pi t)$.

(a) Find the position of the particle when $t = 2.5$.

(b) On the axes provided below, sketch the graph of the path of the particle from $t = 0$ to $t = 6$. Indicate the direction of the particle along its path.

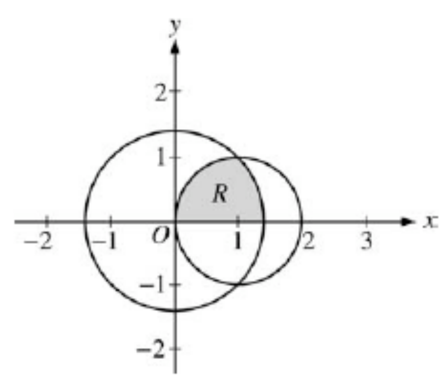
(c) How many times does the particle pass through the point found in part (a)?



(e) Write and evaluate an integral expression, in terms of sine and cosine, that gives the distance the particle travels from $t = 1.25$ to $t = 1.75$.

4. NO CALCULATOR

The figure above shows the graphs of the circles $x^2 + y^2 = 2$ and $(x-1)^2 + y^2 = 1$. The graphs intersect at the points $(1, 1)$ and $(1, -1)$. Let R be the shaded region in the first quadrant bounded by the two circles and the x-axis.



(a) Set up an expression involving one or more integrals with respect to x that represents the area of R.

(b) Set up an expression involving one or more integrals with respect to y that represents the area of R.

(c) The polar equations of the circles are $r = \sqrt{2}$ and $r = 2\cos\theta$, respectively. Set up an expression involving one or more integrals with respect to the polar angle θ that represents the area of R.

5. NO CALCULATOR

Consider the polar curve $r = 2\sin(3\theta)$ for $0 \le \theta \le \pi$.

(a) In the xy-plane provided below, sketch the curve.

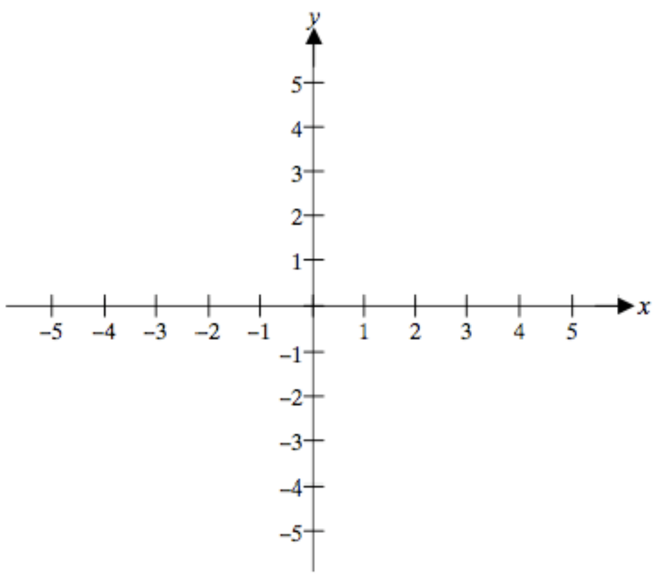
(b) Find the area of the region inside the curve.

(c) Find the slope of the curve at the point where $\theta = \frac{\pi}{4}$.

6. NO CALCULATOR

Let R be the region inside the graph of the polar curve $r = 2$ and outside the graph of the polar curve $r = 2(1 - \sin\theta)$.

(a) Sketch the two polar curves in the xy-plane provided below and shade the region R



(b) Find the area of R .

7. CALCULATOR

Consider the curves $r = 3\cos\theta$ and $r = 1 + \cos\theta$.

(a) Sketch the curves on the same set of axes.

(b) Find the area of the region inside the curve $r = 3\cos\theta$ and outside the curve $r = 1 + \cos\theta$ by setting up and evaluating a definite integral. Your work must include an antiderivative.

8. CALCULATOR

A particle moves on the circle $x^2 + y^2 = 1$ so that at time $t \geq 0$ the position is given by the vector $\left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2} \right)$.

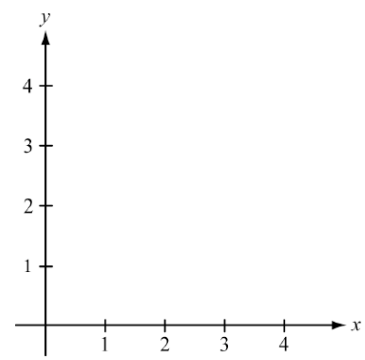
(a) Find the velocity vector.

(b) Is the particle ever at rest? Justify your answer.

(c) Give the coordinates of the point that the particle approaches as t increases without bound.

9. CALCULATOR

A particle moves in the xy-plane so that at any time $t \geq 0$ its position (x, y) is given by $x = e^t + e^{-t}$ and $y = e^t - e^{-t}$.



(a) Find the velocity vector for any $t \geq 0$.

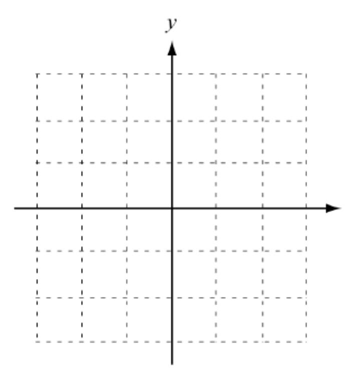
(b) Find $\lim_{t \to \infty} \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.

(c) The particle moves on a hyperbola. Find an equation for this hyperbola in terms of x and y.

(d) On the axes provided, sketch the path of the particle showing the velocity vector for $t = 0$.

10. CALCULATOR

The position of a particle moving in the xy-plane at any time t, $0 \leq t \leq 2\pi$, is given by the parametric equations $x = \sin t$ and $y = \cos(2t)$.



(a) Find the velocity vector for the particle at any time t, $0 \leq t \leq 2\pi$.

(b) For what values of t is the particle at rest?

(c) Write an equation for the path of the particle in terms of x and y that does <u>not</u> involve trigonometric functions.

(d) Sketch the path of the particle in the xy-plane below.

11. CALCULATOR

The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$.

(a) Find the magnitude of the velocity vector at $t = 5$.

(b) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

(c) Find $\frac{dy}{dx}$ as a function of x.

12. CALCULATOR

A particle moves in the xy-plane so that the position of the particle at any time t is given by

$$x(t) = 2e^{3t} + e^{-7t} \text{ and } y(t) = 3e^{3t} - e^{-2t}.$$

(a) Find the velocity vector for the particle in terms of t, and find the speed of the particle at time $t = 0$.

(b) Find $\frac{dy}{dx}$ in terms of t, and find $\lim_{t \to \infty} \frac{dy}{dx}$.

(c) Find each value t at which the line tangent to the path of the particle is horizontal, or explain why none exists.

(d) Find each value t at which the line tangent to the path of the particle is vertical, or explain why none exists.

13. CALCULATOR

A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \text{ and } \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of t. At time $t = 0$, the particle is at the point $(4, 1)$.

(a) Find the speed of the particle and its acceleration vector at time $t = 0$.

(b) Find an equation of the line tangent to the path of the particle at time $t = 0$.

(c) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

(d) Find the x-coordinate of the position of the particle at time $t = 3$.

14. CALCULATOR

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = 12t - 3t^2 \text{ and } \frac{dy}{dt} = \ln\left(1 + (t-4)^4\right).$$

At time $t = 0$, the object is at position $(-13, 5)$. At time $t = 2$, the object is at point P with x-coordinate 3.

(a) Find the acceleration vector at time $t = 2$ and the speed at time $t = 2$.

(b) Find the y-coordinate of P.

(c) Write an equation for the line tangent to the curve at P.

(d) For what value of t, if any, is the object at rest? Explain your reasoning.

15. CALCULATOR

An object moving along a curve in the xy-plane is at position $(x(t),\, y(t))$ at time t, where

$$\frac{dx}{dt} = \tan\left(e^{-t}\right) \text{ and } \frac{dy}{dt} = \sec\left(e^{-t}\right)$$

for $t \geq 0$. At time $t = 1$, the object is at position $(2, -3)$.

(a) Write an equation for the line tangent to the curve at position $(2, -3)$.

(b) Find the acceleration vector and the speed of the object at time $t = 1$.

(c) Find the total distance traveled by the object over the time interval $1 \leq t \leq 2$.

(d) Is there a time $t \geq 0$ at which the object is on the y-axis? Explain why or why not.

16. CALCULATOR

An object moving along a curve in the xy-plane is at position $(x(t),\, y(t))$ at time t with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right) \text{ and } \frac{dy}{dt} = \ln\left(t^2+1\right)$$

for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1}x = \arctan x$)

(a) Find the speed of the object at time $t = 4$.

(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.

(c) Find $x(4)$.

(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

17. CALCULATOR

A particle moving along a curve in the xy-plane has position $(x(t),\, y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = \sqrt{3t} \text{ and } \frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right).$$

The particle is at position $(1, 5)$ at time $t = 4$.

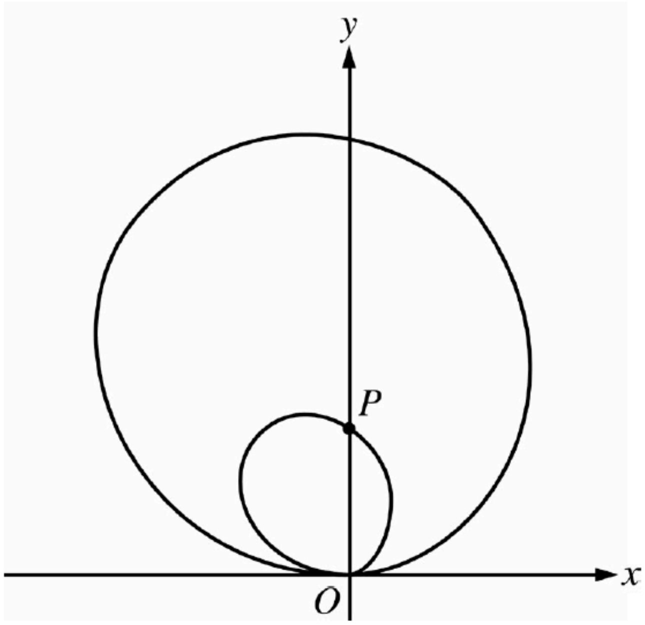
(a) Find the acceleration vector at time $t = 4$.

(b) Find the y-coordinate of the position of the particle at time $t = 0$.

(c) On the interval $0 \leq t \leq 4$, at what time does the speed of the particle first reach 3.5 ?

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 4$.

18.



Let r be the function given by $r(\theta) = 3\theta \sin \theta$ for $0 \leq \theta \leq 2\pi$. The graph of r in polar coordinates consists of two loops, as shown in the figure above. Point P is on the graph of r and the y-axis.

(A) Find the rate of change of the x-coordinate with respect to θ at the point P.

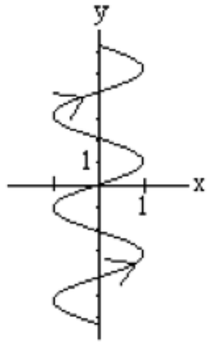
(B) Find the area of the region between the inner and outer loops of the graph.

(C) The function r satisfies $\frac{dr}{d\theta} = 3\sin\theta + 3\theta\cos\theta$. For $0 \leq \theta \leq 2\pi$, find the value of θ that gives the point on the graph that is farthest from the origin. Justify your answer.

Ch 9 AP Problems - ANSWERS

1.

(a)



(b) $-1 \le x(t) \le 1$

$-2\pi \le y(t) \le 2\pi$

(c) $x'(t) = 3\cos 3t = 0$

$3t = \frac{\pi}{2}$; $t = \frac{\pi}{6}$

Speed $= \sqrt{9\cos^2(3t) + 4}$

At $t = \frac{\pi}{6}$,

Speed $= \sqrt{9\cos^2\left(\frac{\pi}{2}\right) + 4} = 2$

(d) Distance $= \int_{-\pi}^{\pi} \sqrt{9\cos^2(3t) + 4}\,dt$

$= 17.973 > 5\pi$

2.

(a) $\frac{dx}{dt} = e^t \sin t + e^t \cos t$

$\frac{dy}{dt} = e^t \cos t - e^t \sin t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^t(\cos t - \sin t)}{e^t(\sin t + \cos t)}$

at $t = \frac{\pi}{2}$, $\frac{dy}{dx} = \frac{e^{\pi/2}(0-1)}{e^{\pi/2}(1+0)} = -1$

(b) speed $= \sqrt{\left(e^t \sin t + e^t \cos t\right)^2 + \left(e^t \cos t - e^t \sin t\right)^2}$

when $t = 1$ speed is

$\sqrt{(e\sin 1 + e\cos 1)^2 + (e\cos 1 - e\sin 1)^2} = e\sqrt{2}$

(c) distance is

$\int_0^1 \sqrt{\left(e^t \sin t + e^t \cos t\right)^2 + \left(e^t \cos t - e^t \sin t\right)^2}\,dt$

$= \int_0^1 \sqrt{2e^{2t}\left(\sin^2 t + \cos^2 t\right)}\,dt = \int_0^1 \sqrt{2}e^t\,dt$

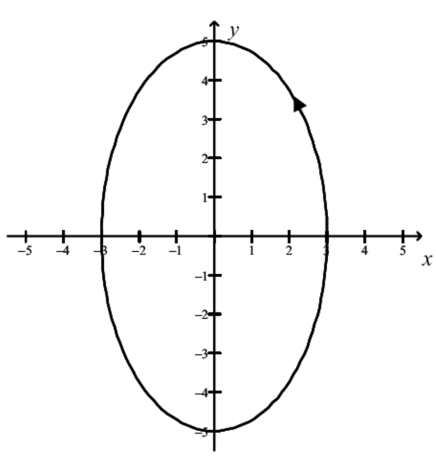
$= \sqrt{2}\,e^t\Big|_0^1 = \sqrt{2}(e-1)$

3.

(a) $x(2.5) = 3\cos(2.5\pi) = 0$

$y(2.5) = 5\sin(2.5\pi) = 5$

(b)



(c) 3

(d) $x'(t) = -3\pi\sin(\pi t) \quad y'(t) = 5\pi\cos(\pi t)$

$\vec{v}(t) = \langle -3\pi\sin(\pi t), 5\pi\cos(\pi t)\rangle$

(e) distance $= \int_{1.25}^{1.75} \sqrt{9\pi^2\sin^2(\pi t) + 25\pi^2\cos^2(\pi t)}\,dt$

$= 5.392$

4.

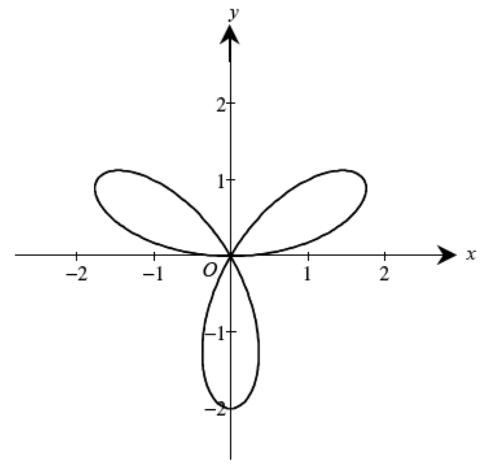
(a) Area $= \int_0^1 \sqrt{1-(x-1)^2}\,dx + \int_1^{\sqrt{2}} \sqrt{2-x^2}\,dx$

(b) Area $= \int_0^1 \left(\sqrt{2-y^2} - \left(1-\sqrt{1-y^2}\right)\right) dy$

(c) Area $= \int_0^{\pi/4} \frac{1}{2}\left(\sqrt{2}\right)^2 d\theta + \int_{\pi/4}^{\pi/2} \frac{1}{2}(2\cos\theta)^2\,d\theta$

5.

(a)



(b) $A = \frac{1}{2}\int_0^{\pi} 4\sin^2 3\theta\,d\theta = \int_0^{\pi}(1-\cos 6\theta)\,d\theta = \left[\theta - \frac{1}{6}\sin 6\theta\right]_0^{\pi} = \pi$

or $\frac{3}{2}\int_0^{\pi/3} 4\sin^2 3\theta\,d\theta = \cdots = \pi$

or $\frac{6}{2}\int_0^{\pi/6} 4\sin^2 3\theta\,d\theta = \cdots = \pi$

(c) $x = 2\sin 3\theta\cos\theta$

$y = 2\sin 3\theta \sin\theta$

$\frac{dx}{d\theta} = -2\sin 3\theta\sin\theta + 6\cos 3\theta\cos\theta$

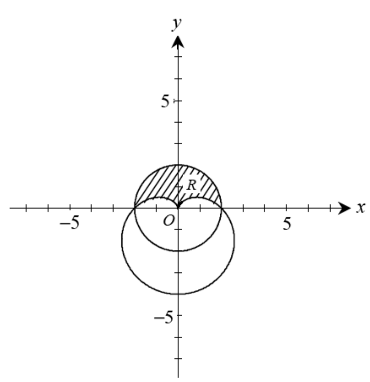
$\frac{dy}{d\theta} = 2\sin 3\theta\cos\theta + 6\cos 3\theta\sin\theta$

At $\theta = \frac{\pi}{4}$, $\frac{dy}{d\theta} = -2$ and $\frac{dx}{d\theta} = -4$, so

$\frac{dy}{dx} = \frac{-2}{-4} = \frac{1}{2}$

6.

(a)

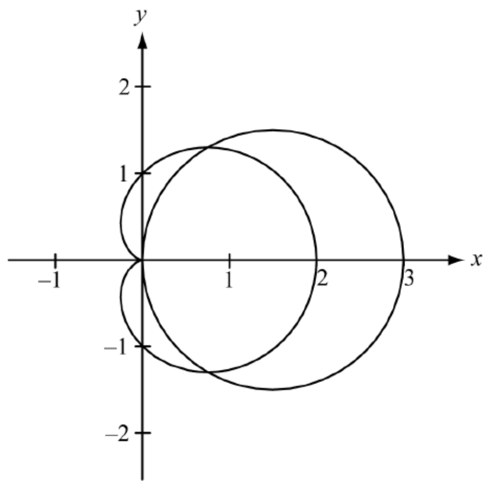


(b)

$$A = \frac{1}{2}\int_0^{\pi}\left[2^2 - \left(2\left(1-\sin\theta\right)\right)^2\right]d\theta$$
$$= 2\int_0^{\pi}\left(2\sin\theta - \sin^2\theta\right)d\theta$$
$$= 4\int_0^{\pi}\sin\theta\,d\theta - \int_0^{\pi}\left(1-\cos 2\theta\right)d\theta$$
$$= -4\cos\theta\Big|_0^{\pi} - \left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi}$$
$$= \left[-4(-1)+4(1)\right] - \left[\pi - 0\right]$$
$$= 8-\pi$$

7.

(a)



(b) The intersection occurs when $3\cos\theta = 1+\cos\theta$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \pm\frac{\pi}{3}$$

$$\begin{aligned}
\text{Area } &= \frac{1}{2}\int_{-\pi/3}^{\pi/3}\left((3\cos\theta)^2 - (1+\cos\theta)^2\right)d\theta \\
&= \frac{1}{2}\int_{-\pi/3}^{\pi/3}(8\cos^2\theta - 1 - 2\cos\theta)\,d\theta \\
&= \frac{1}{2}\int_{-\pi/3}^{\pi/3}(4+4\cos 2\theta - 1 - 2\cos\theta)\,d\theta \\
&= \frac{1}{2}\int_{-\pi/3}^{\pi/3}(3+4\cos 2\theta - 2\cos\theta)\,d\theta \\
&= \frac{1}{2}(3\theta + 2\sin 2\theta - 2\sin\theta)\Big|_{-\pi/3}^{\pi/3} \\
&= \frac{1}{2}\left(\pi + 2\sin\frac{2\pi}{3} - 2\sin\frac{\pi}{3} - (-\pi - 2\sin\frac{2\pi}{3} + \sin\frac{\pi}{3})\right) \\
&= \pi
\end{aligned}$$

8.

(a) $$\frac{dx}{dt} = \frac{(1+t^2)(-2t)-(1-t^2)(2t)}{(1+t^2)^2} = \frac{-4t}{(1+t^2)^2}$$

$$\frac{dy}{dt} = \frac{(1+t^2)(2)-2t(2t)}{(1+t^2)^2} = \frac{2-2t^2}{(1+t^2)^2}$$

The velocity vector is $\left(\dfrac{-4t}{(1+t^2)^2}, \dfrac{2-2t^2}{(1+t^2)^2}\right)$

(b) $\dfrac{dx}{dt} = 0$ at $t=0$ and $\dfrac{dy}{dt} = 0$ at $t=1$. Therefore there is no t such that $\dfrac{dx}{dt} = \dfrac{dy}{dt} = 0$ at the same time. Hence the particle is never at rest.

(c) $$\lim_{t\to\infty} x(t) = \lim_{t\to\infty}\frac{1-t^2}{1+t^2} = \lim_{t\to\infty}\frac{\frac{1}{t^2}-1}{\frac{1}{t^2}+1} = -1$$

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty}\frac{2t}{1+t^2} = \lim_{t\to\infty}\frac{2}{\frac{1}{t}+t} = 0$$

Hence the particle approaches the point $(-1, 0)$ as t increases without bound.

9.

(a) $$\frac{dx}{dt} = e^t - e^{-t} \qquad \frac{dy}{dt} = e^t + e^{-t}$$

$$\vec{v}(t) = (e^t - e^{-t}, e^t + e^{-t}) = (e^t - e^{-t})\vec{i} + (e^t + e^{-t})\vec{j}$$

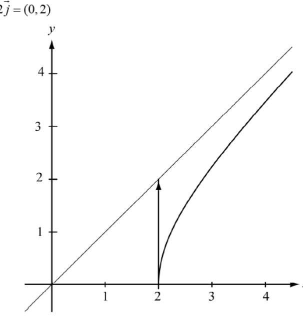
(b) <u>Method 1</u>

$$\lim_{t\to\infty}\frac{1+\frac{1}{e^{2t}}}{1-\frac{1}{e^{2t}}} = 1$$

(c) $$\begin{cases} x^2 = e^{2t} + 2 + e^{-2t} \\ y^2 = e^{2t} - 2 + e^{2t} \end{cases}$$

Therefore $x^2 - y^2 = 4$

(d) $\vec{v}(0) = 2\vec{j} = (0,2)$



10.

(a) $x = \sin(t),\ y = \cos(2t)$

$$\frac{dx}{dt} = \cos(t),\ \frac{dy}{dt} = -2\sin(2t)$$

$$\vec{v} = (\cos(t), -2\sin(2t))$$

(b) $\vec{v} = 0 \Rightarrow \cos(t) = 0$ and $-2\sin(2t) = 0$

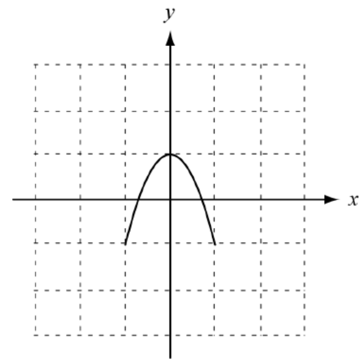
Therefore $\cos(t) = 0$ and $-4\sin(t)\cos(t) = 0$. The only choice is $\cos(t) = 0$.

Therefore the particle is at rest when $t = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$.

(c) $x^2 = \sin^2(t),\ \ y = \cos^2(t) = 1 - 2\sin^2(t)$

$y = 1 - 2x^2$

(d)



11.

(a) $x'(t) = 2t \qquad y'(t) = 2t^2$

$x'(5) = 10 \qquad y'(5) = 50$

$$\|v(5)\| = \sqrt{10^2 + 50^2} = \sqrt{2600}$$

$$= 10\sqrt{26} \approx 50.990$$

(b) $\displaystyle\int_0^5 \sqrt{4t^2 + 4t^4}\, dt$

$$= \int_0^5 2t\sqrt{1 + t^2}\, dt$$

$$= \frac{2}{3}\left(1 + t^2\right)^{3/2}\Big|_0^5$$

$$= \frac{2}{3}\left(26^{3/2} - 1\right) \approx 87.716$$

(c) $\dfrac{dy}{dx} = \dfrac{y'(t)}{x'(t)} = \dfrac{2t^2}{2t} = t$

$x = t^2 - 3;\ \ t^2 = x + 3$

$t = \sqrt{x+3}$

$$\frac{dy}{dx} = \sqrt{x+3}$$

12.

(a) $x'(t) = 6e^{3t} - 7e^{-7t}$

$y'(t) = 9e^{3t} + 2e^{-2t}$

Velocity vector is $< 6e^{3t} - 7e^{-7t},\ 9e^{3t} + 2e^{-2t} >$

Speed $= \sqrt{x'(0)^2 + y'(0)^2} = \sqrt{(-1)^2 + 11^2}$

$= \sqrt{122}$

(b) $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}}$$

$$\lim_{t\to\infty} \frac{dy}{dx} = \lim_{t\to\infty} \frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}} = \frac{9}{6} = \frac{3}{2}$$

(c) Need $y'(t) = 0$, but $9e^{3t} + 2e^{-2t} > 0$ for all t, so none exists.

(d) Need $x'(t) = 0$ and $y'(t) \neq 0$.

$6e^{3t} = 7e^{-7t}$

$e^{10t} = \dfrac{7}{6}$

$t = \dfrac{1}{10}\ln\left(\dfrac{7}{6}\right)$

13.

(a) At time $t = 0$:

Speed $= \sqrt{x'(0)^2 + y'(0)^2} = \sqrt{3^2 + 7^2} = \sqrt{58}$

Acceleration vector $= \langle x''(0), y''(0)\rangle = \langle 0, -3\rangle$

(b) $\dfrac{dy}{dx} = \dfrac{y'(0)}{x'(0)} = \dfrac{7}{3}$

Tangent line is $y = \frac{7}{3}(x-4)+1$

(c) Distance $= \int_0^3 \sqrt{\left(\sqrt{t^4+9}\right)^2 + \left(2e^t + 5e^{-t}\right)^2}\, dt$

$= 45.226$ or 45.227

(d) $x(3) = 4 + \int_0^3 \sqrt{t^4+9}\, dt$

$= 17.930$ or 17.931

14.

(a) $x''(2) = 0,\ y''(2) = -\dfrac{32}{17} = -1.882$

$a(2) = \langle 0, -1.882\rangle$

Speed $= \sqrt{12^2 + (\ln(17))^2} = 12.329$ or 12.330

(b) $y(t) = y(0) + \int_0^t \ln\left(1 + (u-4)^4\right) du$

$y(2) = 5 + \int_0^2 \ln\left(1 + (u-4)^4\right) du = 13.671$

(c) At $t = 2$, slope $= \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{\ln(17)}{12} = 0.236$

$y - 13.671 = 0.236(x-3)$

(d) $x'(t) = 0$ if $t = 0, 4$

$y'(t) = 0$ if $t = 4$

$t = 4$

15.

(a) $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{\sec\left(e^{-t}\right)}{\tan\left(e^{-t}\right)} = \dfrac{1}{\sin\left(e^{-t}\right)}$

$\left.\dfrac{dy}{dx}\right|_{(2,-3)} = \dfrac{1}{\sin\left(e^{-1}\right)} = 2.780$ or 2.781

$y + 3 = \dfrac{1}{\sin\left(e^{-1}\right)}(x-2)$

(b) $x''(1) = -0.42253,\ y''(1) = -0.15196$

$a(1) = \langle -0.423, -0.152\rangle$ or $\langle -0.422, -0.151\rangle$.

speed $= \sqrt{\left(\sec\left(e^{-1}\right)\right)^2 + \left(\tan\left(e^{-1}\right)\right)^2} = 1.138$ or 1.139

(c) $\int_1^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 1.059$

(d) $x(0) = x(1) - \int_0^1 x'(t)\, dt = 2 - 0.775553 > 0$

The particle starts to the right of the y-axis.
Since $x'(t) > 0$ for all $t \ge 0$, the object is always moving to the right and thus is never on the y-axis.

16.

(a) Speed $= \sqrt{x'(4)^2 + y'(4)^2} = 2.912$

(b) Distance $= \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 6.423$

(c) $x(4) = x(0) + \int_0^4 x'(t)\,dt$

$= -3 + 2.10794 = -0.892$

(d) The slope is 2, so $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2$, or $\ln\left(t^2+1\right) = 2\arctan\left(\frac{t}{1+t}\right)$.

Since $t > 0$, $t = 1.35766$. At this time, the acceleration is $\langle x''(t), y''(t)\rangle\big|_{t=1.35766} = \langle 0.135, 0.955\rangle$.

17.

(a) $a(4) = \langle x''(4), y''(4)\rangle = \langle 0.433, -11.872\rangle$

(b) $y(0) = 5 + \int_4^0 3\cos\left(\frac{t^2}{2}\right) dt = 1.600$ or 1.601

(c) Speed $= \sqrt{(x'(t))^2 + (y'(t))^2}$

$= \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)} = 3.5$

The particle first reaches this speed when $t = 2.225$ or 2.226.

(d) $\int_0^4 \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)}\, dt = 13.182$

18.

(a) $x = r\cos\theta = 3\theta\sin\theta\cos\theta$

At point P, $\theta = \frac{\pi}{2}$.

$\left.\frac{dx}{d\theta}\right|_{\theta=\pi/2} = -4.712$

(B) Area $= \frac{1}{2}\int_\pi^{2\pi} (r(\theta))^2\, d\theta - \frac{1}{2}\int_0^{\pi} (r(\theta))^2\, d\theta = 139.528$

(C) $3\sin\theta + 3\theta\cos\theta = 0 \Rightarrow \theta = 2.028758,\ \theta = 4.913180$

θ	$r(\theta)$
0	0
2.028758	5.459117
4.913180	−14.443410
2π	0

The value $\theta = 4.913$ gives the point on the graph that is farthest from the origin.