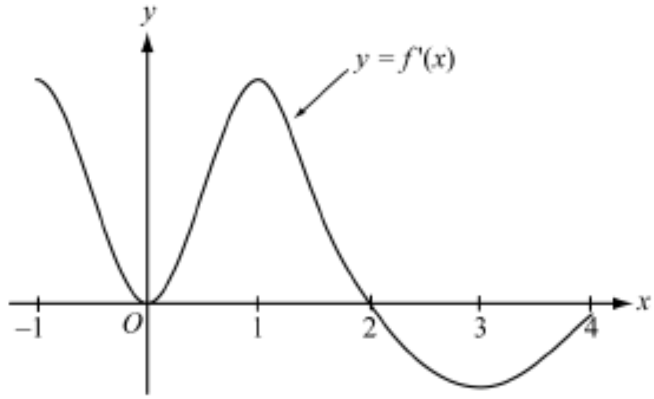
Chapter 6-7 Test Review Topics

1) Interpreting f' graphs



Note: This is the graph of the derivative of f, NOT the graph of f.

Let f be a function that has domain the closed interval $[-1,4]$ and range the closed interval $[-1,2]$. Let $f(-1)=-1$, $f(0)=0$, and $f(4)=1$. Also let f have the derivative function f' that is continuous and that has the graph shown in the figure above.

(a) Find all values of x for which f assumes a relative maximum. Justify your answer.

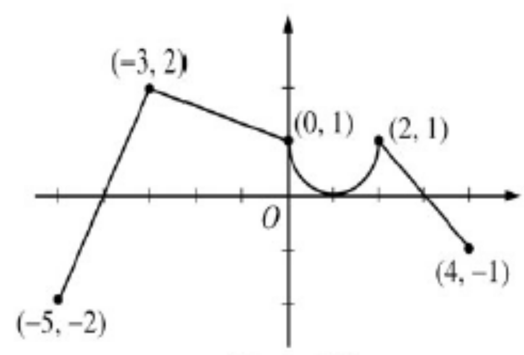
(b) Find all values of x for which f assumes its absolute minimum. Justify your answer.

(c) Find the intervals on which f is concave downward.

(d) Give all the values of x for which f has a point of inflection.

2) Accumulation Functions (2nd FTC)

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

3) Motion on a line

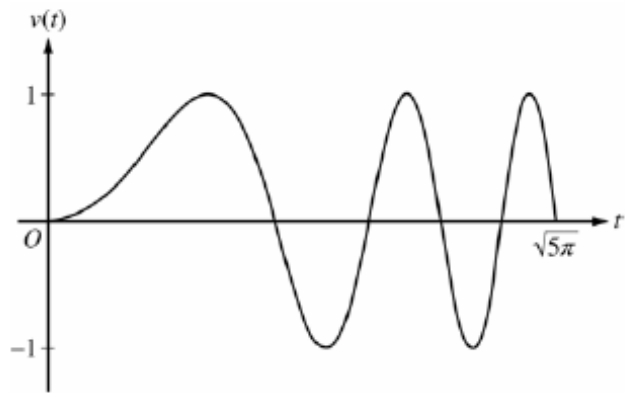
A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.



4) Tabular Data

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	–20	–30	–20	–14	–10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \leq t \leq 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\,dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\,dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\,dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\,dt$.

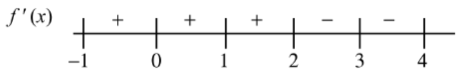
(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

Answers

1)

(a) $f'(x) = 0$ at $x = 0, 2$



There is a relative maximum at $x = 2$, since $f'(2) = 0$ and $f'(x)$ changes from positive to negative at $x = 2$.

(b) There is no minimum at $x = 0$, since $f'(x)$ does not change sign there. So the absolute minimum must occur at an endpoint. Since $f(-1) < f(4)$, the absolute minimum occurs at $x = -1$.

(c) The graph of f is concave down on the intervals $[-1, 0)$ and $(1, 3)$ because f' is decreasing on those intervals.

(d) The graph of f has a point of inflection at $x = 0$, 1, and 3 because f' changes from decreasing to increasing or from increasing to decreasing at each of those x values.

2)

(a) $g(0) = \int_{-3}^{0} f(t)\,dt = \frac{1}{2}(3)(2+1) = \frac{9}{2}$

$g'(0) = f(0) = 1$

(b) g has a relative maximum at $x = 3$.
This is the only x-value where $g' = f$ changes from positive to negative.

(c) The only x-value where f changes from negative to positive is $x = -4$. The other candidates for the location of the absolute minimum value are the endpoints.

$g(-5) = 0$

$g(-4) = \int_{-3}^{-4} f(t)\,dt = -1$

$g(4) = \frac{9}{2} + \left(2 - \frac{\pi}{2}\right) = \frac{13 - \pi}{2}$

So the absolute minimum value of g is -1.

(d) $x = -3, 1, 2$

3)

(a) $a(3) = v'(3) = 6\cos 9 = -5.466$ or -5.467

(b) Distance $= \int_0^3 |v(t)|\, dt = 1.702$

OR

For $0 < t < 3$, $v(t) = 0$ when $t = \sqrt{\pi} = 1.77245$ and

$t = \sqrt{2\pi} = 2.50663$

$x(0) = 5$

$x(\sqrt{\pi}) = 5 + \int_0^{\sqrt{\pi}} v(t)\, dt = 5.89483$

$x(\sqrt{2\pi}) = 5 + \int_0^{\sqrt{2\pi}} v(t)\, dt = 5.43041$

$x(3) = 5 + \int_0^3 v(t)\, dt = 5.77356$

$|x(\sqrt{\pi}) - x(0)| + |x(\sqrt{2\pi}) - x(\sqrt{\pi})| + |x(3) - x(\sqrt{2\pi})| = 1.702$

(c) $x(3) = 5 + \int_0^3 v(t)\, dt = 5.773$ or 5.774

(d) The particle's rightmost position occurs at time $t = \sqrt{\pi} = 1.772$.

The particle changes from moving right to moving left at those times t for which $v(t) = 0$ with $v(t)$ changing from positive to negative, namely at $t = \sqrt{\pi}, \sqrt{3\pi}, \sqrt{5\pi}$ $(t = 1.772, 3.070, 3.963)$.

Using $x(T) = 5 + \int_0^T v(t)\, dt$, the particle's positions at the times it changes from rightward to leftward movement are:

T:	0	$\sqrt{\pi}$	$\sqrt{3\pi}$	$\sqrt{5\pi}$
$x(T)$:	5	5.895	5.788	5.752

The particle is farthest to the right when $T = \sqrt{\pi}$.

4)

(a) $\int_{30}^{60} |v(t)|\, dt$ is the distance in feet that the car travels from $t = 30$ sec to $t = 60$ sec.

Trapezoidal approximation for $\int_{30}^{60} |v(t)|\, dt$:

$$A = \frac{1}{2}(14 + 10)5 + \frac{1}{2}(10)(15) + \frac{1}{2}(10)(10) = 185 \text{ ft}$$

(b) $\int_0^{30} a(t)\, dt$ is the car's change in velocity in ft/sec from $t = 0$ sec to $t = 30$ sec.

$$\int_0^{30} a(t)\, dt = \int_0^{30} v'(t)\, dt = v(30) - v(0)$$
$$= -14 - (-20) = 6 \text{ ft/sec}$$

(c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.

(d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.

Units of ft in (a) and ft/sec in (b)