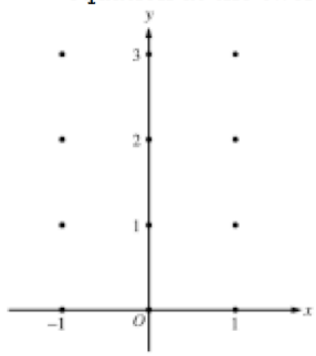
AP Calculus - Chapter 5/6 FR Review

1. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$ (no calculator)

a. On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

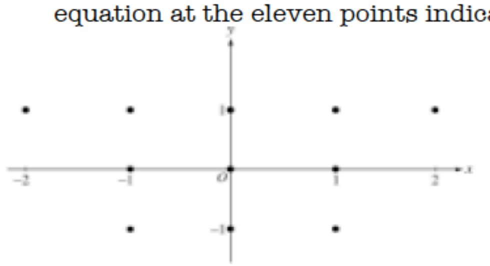


b. Find the particular solution y = f(x) to the given differential equation with the initial condition f(0) = 3.

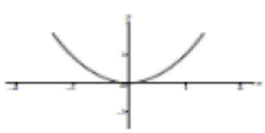
c. Write an equation of the line tangent to y = f(x) at x = 2.

2. Consider the differential equation given by $\frac{dy}{dx} = x(y-1)^2$ (no calculator)

a. On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.



b. Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.



c. Find the particular solution y = f(x) to the given differential equation with the initial condition f(0) = -1

d. Find the range of the solution found in part (c).

3. (no calculator)

Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

(a) Find the area of the region R.

(b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

(c) Find the volume of the solid generated when R is revolved about the x–axis.

(d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

4. (calculator)

. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$.

(a) Find the slope of the graph of f at the point where $x = 1$.

(b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

(c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.

(d) Use your solution from part (c) to find $f(1.2)$.

5. (calculator)

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

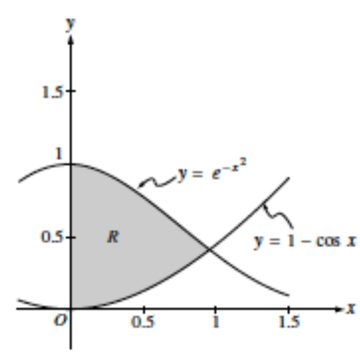
6. (calculator)

Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

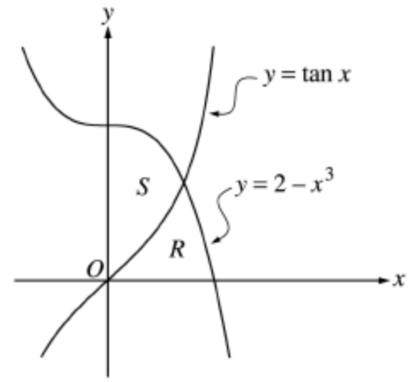
(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



7. (calculator)

Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



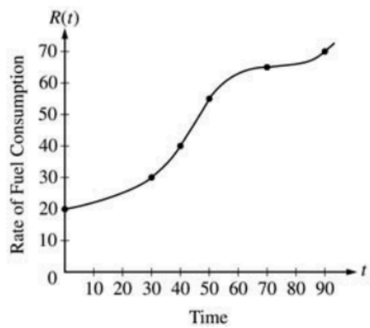
(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.

8. (no calculator)

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

Answers

1b	$y = 3e^{x^2/4}$
1c	y – 3e = 3e(x – 2)
2b	There is a horizontal asymptote at y = 1
2c	$y = \frac{-2}{x^2+1} + 1$
2d	range: [-1, 1)
3a	16/3
3b	$4^{2/3}$
3c	8π
3d	$2\sqrt{2}$
4a	½
4b	4.1
4c	$f(x) = \sqrt{x^3 + x + 14}$
4d	4.114
5a	6004 people entered the park by 5 pm
5b	The amount collected was $104,048
5c	There were 375 people in the park at t=17. Since $H'(17) = E(17) - L(17) = -380.281 < 0$, the number of people in the park was decreasing at a rate of approximately 380 people/hr at t=17 hrs.
5d	t=15.794 hrs by the Extreme Value Theorem
6a	0.590 or 0.591
6b	1.746 or 1.747
6c	0.461
7a	0.729
7b	1.160 or 1.161
7c	8.331 or 8.332
8a	1.5 gal/min^2
8b	R''(45)=0 since R'(t) has a maximum at t=45
8c	3700; Yes, this approximation is less because the graph of R is increasing on the interval
8d	$\int_0^b R(t)\,dt$ is the total amount of fuel in gallons consumed for the first b minutes. $\frac{1}{b}\int_0^b R(t)\,dt$ is the average value of the rate of fuel consumption in gallons/min during the first b minutes.